

This is a very thorough and carefully-written text. It should challenge Hatcher's text for the high-end market; i.e., for use at major universities. The approach is more formal than Hatcher's, more of a theorem-proof exposition than his. There are very few pictures, and not many illustrative examples, e.g., of covering spaces or homology calculations. There are, however, many challenging exercises, called "problems."

The first 220 pages are homotopy theory. In addition to the usual fundamental group, covering spaces, (co)fibrations, homotopy groups, and cell complexes, this also includes an excellent chapter on stable homotopy and duality. The stable category used here has as objects pairs (X, n) with X a pointed space and $n \in \mathbb{Z}$, and as morphisms stable homotopy classes of maps. For duality, it is shown that, for suitable X , if Y is the mapping cone of $\mathbb{R}^n - X \rightarrow \mathbb{R}^n$, then there are duality maps $Y \wedge X^+ \rightarrow S^n$ and $S^n \rightarrow X^+ \wedge Y$. This chapter also includes brief sections on generalized (co)homology and spectral (co)homology.

Next we have 100 pages on singular and cellular homology. The usual applications of homology are included here. Classification of surfaces is mentioned only very briefly here and again later in the book. A nice touch here is a separate chapter on homological algebra.

The next section (90 pages) is bundles and manifolds. This includes, among other things, Milnor's classifying space construction and the definition of $K(X)$, with a brief mention of the Bott periodicity theorem.

Finally on page 405 we get to cohomology, with 90 pages including the usual Poincaré-type duality theorems and a fairly detailed chapter on characteristic classes (Chern, Stiefel-Whitney, and Pontrjagin). This section of the book also includes the relationship of cohomology with Eilenberg-MacLane spaces, and introduces the signature of the intersection pairing.

The 26-page Chapter 20, entitled "Homology and Homotopy," is quite amazing. It presents, and proves, most of the consequences of Serre's mod \mathcal{C} theory without using spectral sequences. This includes the finiteness of the homotopy groups of spheres, and relationships between finiteness (resp. finite generation) of homology and homotopy groups of a space.

The last chapter, "Bordism," presents a rather general form of the Pontrjagin-Thom theorem relating cobordism classes of manifolds and homotopy groups of a Thom space. The book closes triumphantly with a one-paragraph proof of the Hirzebruch signature theorem (using many previously-proved results).

The exposition is very sophisticated throughout. The text will be difficult for many beginning readers. For example, the Seifert-van Kampen Theorem is stated and proved as a theorem about a pushout of fundamental groupoids. This is then used to prove that $\pi_1(S^1)$ is infinite cyclic by showing that the fundamental groupoid of S^1 is isomorphic to a certain topological groupoid which is explicitly defined.

As another example of the sophisticated treatment, the classification of covering spaces over B is studied by relating the category COV_B of coverings over B to the transport category $\text{TRA}_B = [\Pi(B), \text{SET}]$, whose objects are functors from the fundamental groupoid $\Pi(B)$ to the category of sets. Several times it is mentioned that the rather categorical treatment is consistent with the recent emphasis on model category theory.

There is some non-standard notation, which takes a little getting used to. One nice one is using S^n , $S(n)$, and $S^{(n)}$ for, respectively, the unit sphere, $I^n/\partial I^n$, and

$\mathbb{R}^n \cup \{\infty\}$. Another is $A|B$ for the pair $(A, A - B)$. This gets a little confusing when it appears in $A|B \times C|D$, but is probably worthwhile.

In a superficial reading of the book, only a very few minor typographical errors were noticed. This book has been prepared very carefully. One thing which rankled this reviewer is the treatment of the Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem. For the Brouwer FPT, a theorem (6.6.1) is stated and proved that “the following (nine) statements are equivalent.” The nine include (i) the Brouwer FPT, and (iii) the Homotopy Theorem, that the identity map of S^{n-1} is not null homotopic. After proving the equivalence of the nine statements, it says “Theorem 6.6.1 has many different proofs,” and refers to two references. The intent is clearly that 6.6.1 means that the nine statements are equivalent and true. It is never explicitly stated that the statements are true, although in 6.4.7 it was proved that $\pi_n(S(n)) \cong \mathbb{Z}$. A similar oversight was made in the discussion of the Borsuk-Ulam Theorem.

These quibblings aside, I believe that the best students at the best universities will find this book to be the best path to learning modern algebraic topology. The book will also serve as an excellent reference for many mathematicians.