

I was very fortunate to have a 40-year collaboration with Mark Mahowald, resulting in 35 published papers, with publication dates extending from 1975 to 2014. He taught me more about homotopy theory than I will ever know.

My Stanford thesis advisor, Jim Milgram, knew that Mark and I would be a good pair, and they arranged for my postdoctoral position at Northwestern in 1972-4. My thesis was related to the problem of finding the smallest Euclidean space in which real projective n -space could be immersed, and Mark was a top expert on this question.

In my first meeting with Mark, he asked me what I was working on. I explained a certain approach to the immersion problem (that never amounted to much) and he suggested a totally different approach, which I immediately began to study. It involved an inductive argument, largely numerical but with a lot of underlying topology to justify the induction. One part of it resulted in our 1975 paper ([2]), from which we were able to deduce many immersion results in subsequent papers. We were never able to prove the part that he really wanted, one with which he was quite obsessed over the years. The best we were able to do was in a 2008 paper ([4]); with some very hard work we found a tiny improvement on all known results. In the end, we needed a compatibility condition that he felt should be satisfied, but he could never convince me that it was provable.

This was very typical of our work. He had enormous insight, but needed people like me to pin it down.

One time I didn't do a good enough job. In a 1982 paper ([3]), we proved that there could not be a spectrum with certain cohomology groups. In the late 1990's Haynes Miller and Mike Hopkins introduced some new spectra and, together with Mark, discovered that one of them had precisely the cohomology that we had shown to be not realizable. With great effort, Mark eventually realized that the mistake in our work had come from an error in the published information about the homotopy groups of spheres. Ironically, Mark and I were then able to use this new spectrum, tmf , to obtain some new nonimmersion results for real projective spaces.([1])

Working with Mark was great fun. We shared a common interest in sports, which we enjoyed discussing. Unfortunately, the first time he took me sailing I got seasick, and never went again. We spent the fall of 1982 at University of Warwick with our wives. I bought a car, and we frequently took the Mahowalds on weekend road trips. This and a 1990 stay together in Oxford were great experiences for me, thanks to Mark. In each case, he was invited and then managed to extend the invitation to include me.

I visited Northwestern many times to work with Mark, usually staying with him and Zoe. I very much appreciated their hospitality. I also spent 1978-9 as Visiting Associate Professor at Northwestern, with my family. That was possibly the coldest snowiest winter ever in Chicago. But a new 20-inch snowfall on top of 20 that had already fallen did not deter Mark from driving me and the rest of the NU topology group to the Midwest Topology Seminar at University of Chicago. We were about the only car on the road. This was Mark; nothing could stop him.

REFERENCES

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- [2] D. M. Davis and M. Mahowald, *The geometric dimension of some vector bundles over projective spaces*, Trans Amer Math Soc **205** (1975) 295–316.
- [3] ———, *The nonrealizability of the quotient $A//A_2$ of the Steenrod algebra*, Amer Jour Math **04** (1982) 1211–1216.
- [4] G. Dula, D. M. Davis, J. Gonzalez, and M. Mahowald, *Immersions of RP^{2^e-1}* , Alg and Geom Topology **8** (2008) 997–1030.