Contents

Preface

In this book, we introduce the reader to some beautiful and interesting mathematics, which is not only historically important but also still very much alive today. Indeed, it plays a central role in modern mathematics.

The mathematical content of this book is outlined in the introduction, but we shall preview it here. It is a basic property of the integers, known as the Fundamental Theorem of Arithmetic, that every integer can be factored into a product of primes in an essentially unique way. Our principal objective in this book is to investigate somewhat more general but still relatively concrete systems (known as rings of integers in quadratic fields) and see when this property does or does not hold for them. We accomplish this objective in Chapters 1 and 2. But this investigation naturally leads us into further investigations—mathematics is like that—and we consider related questions in Chapters 3 and 4, where we investigate the Gaussian integers and Pell's equation, respectively.

The questions we investigate here were at the roots of the development of algebraic number theory. In Chapter 5 we provide an overview of algebraic number theory with emphasis on how the results for quadratic fields generalize to arbitrary algebraic number fields.

We envision several ways in which this book can be used. One way is for a first course in number theory. In our investigations, we begin at the beginning, so this book is suitable for that purpose. Indeed, one of the themes of this book is that one can go a long ways with only elementary methods. To be sure, the topics covered here are not the traditional topics for a first course in number theory (though there is considerable overlap), but there is no reason that the traditional topics need be sacrosanct.

Another way to use this book is for a more advanced course in number theory, and there is plenty of appropriate material here for such a course. Indeed, there is far more than a semester's worth of material here, even for an advanced course.

In this regard, we call the reader's attention to Appendices A and B, on mathematical induction and congruences, respectively. If this book is used as a text for a first course, much of the material in these two appendices should be covered. If this book is used as a text for a more advanced course, these appendices will serve as background.

We have not tried to write a textbook on algebraic number theory in Chapter 5, but rather to provide an overview of the field. But we feel that this overview can serve as a valuable introduction to, and guide for, the student who wishes to study this field, and can also serve as a concrete reference for some of the general results that a student of this field will encounter.

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