Weintraub, Steven H.
Linear algebra for the young mathematician. (English) Zbl 1446.15001
Pure and Applied Undergraduate Texts 42. Providence, RI: American Mathematical Society (AMS) (ISBN 978-1-4704-5084-7/hbk; 978-1-4704-5378-7/ebook). xii, 389 p. (2019).

The book is a new attempt at teaching students linear algebra. It is intended for beginners in mathematics, who want to have a rigorous and mathematical approach to the field of linear algebra. I also think it is a rather bold task to write a linear algebra text in these days, since there are so many texts out there.

So, let us take a look at Steven H. Weintraub's book, and see what it has to offer. First, as an introduction, I think it is always difficult to figure out the correct level of generality. Do you stick with the real numbers as the scalar field? Do you introduce abstract modules over rings right away. Well, this book starts out with an arbitrary field $\mathbb{F}$ and studies $\mathbb{F}^n$ right away. This is slightly different than other basic textbooks, which start out with linear equations. However, it is clear from the very beginning that this book is more for people who want to understand linear algebra rather than just solve equations.

This book, after introducing the vector space $\mathbb{F}^n$, presents linear equations and matrices, and, in Chapter 3, proceeds to abstract vector spaces. Aside from standard material, the author introduces the dual space, determinant, eigenvalues, the Jordan form, bilinear forms, and inner products.

In order to understand a little more about this book, let us have a look at some of the later chapters.

Determinants are explained in Chapter 6. One aspect I think will be appreciated by many students using this book is its introduction to determinants through volume functions. Other books take an axiomatic determinant function approach at the start, which I also like being more algebraically minded, but most students tend to like some geometric interpretation.

Chapters 7 and 8 cover diagonalizabiltiy and the Jordan canonical form. I think these chapters are pretty well done. Part of the development of the Jordan form uses something called an eigenstructure picture, which I think is interesting. In these two chapters, for a future printing, I would recommend to the author to replace the use of the term "hypothesis (S)" by its meaning "splits into linear factors over $\mathbb{F}$", as the latter is not much longer and much more transparent.

The final two chapters (9 and 10) cover forms and inner product spaces, giving the students a fairly comprehensive introduction to linear algebra.

Each chapter has many exercises at its end.

In my opinion, this is a very interesting and well-written approach to linear algebra. It is not the most succinct book, but rather takes its time and attempts to convey to the student ideas and connections to other areas of mathematics. The ideal audience for this book would be students in a first course of linear algebra for mathematicians who need to see proofs. If the students have already had a more computational course in linear algebra, later chapters of this book could be used for a second course covering more theoretical material, such as Jordan forms and bilinear forms.

Reviewer: Jason Polak (Montréal)

MSC:

15-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to linear algebra
15Axx Basic linear algebra
15Bxx Special matrices

Keywords:

linear algebra; determinants; Jordan form; bilinear form; matrices