

CONTENTS

Preface	vii
1 Vector spaces and linear transformations	1
1.1 Basic definitions and examples	1
1.2 Basis and dimension	8
1.3 Dimension counting and applications	17
1.4 Subspaces and direct sum decompositions	22
1.5 Affine subspaces and quotient spaces	24
1.6 Dual spaces	30
2 Coordinates	41
2.1 Coordinates for vectors	42
2.2 Matrices for linear transformations	43
2.3 Change of basis	46
2.4 The matrix of the dual	53
3 Determinants	57
3.1 The geometry of volumes	57
3.2 Existence and uniqueness of determinants	65
3.3 Further properties	68
3.4 Integrality	74
3.5 Orientation	78
3.6 Hilbert matrices	86
4 The structure of a linear transformation I	89
4.1 Eigenvalues, eigenvectors, and generalized eigenvectors . .	91
4.2 Some structural results	97
4.3 Diagonalizability	102
4.4 An application to differential equations	104

5	The structure of a linear transformation II	109
5.1	Annihilating, minimum, and characteristic polynomials . . .	111
5.2	Invariant subspaces and quotient spaces	116
5.3	The relationship between the characteristic and minimum polynomials	119
5.4	Invariant subspaces and invariant complements	122
5.5	Rational canonical form	132
5.6	Jordan canonical form	136
5.7	An algorithm for Jordan canonical form and Jordan basis .	140
5.8	Field extensions	157
5.9	More than one linear transformation	159
6	Bilinear, sesquilinear, and quadratic forms	165
6.1	Basic definitions and results	165
6.2	Characterization and classification theorems	170
6.3	The adjoint of a linear transformation	184
7	Real and complex inner product spaces	189
7.1	Basic definitions	189
7.2	The Gram-Schmidt process	196
7.3	Adjoint, normal linear transformations, and the spectral theorem	202
7.4	Examples	211
7.5	The singular value decomposition	219
8	Matrix groups as Lie groups	223
8.1	Definition and first examples	223
8.2	Isometry groups of forms	224
A	Polynomials	231
A.1	Basic properties	231
A.2	Unique factorization	236
A.3	Polynomials as expressions and polynomials as functions .	239
B	Modules over principal ideal domains	241
B.1	Definitions and structure theorems	241
B.2	Derivation of canonical forms	242
	Bibliography	245
	Index	247
	About the Author	251