## Optical and Photonic Glasses

# Lecture 14:

# Optical Properties Continued – Refraction of Light in Absorbing Materials


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## **Refraction** of light in **absorbing materials**

From Maxwell's theory of electromagnetism, the dielectric permittivity of a material i is related to its magnetic permeability and the speed of light in the medium:

$$c_i = (\varepsilon_i \mu_i)^{-1/2}$$

In non-magnetic media, where $\mu_i \sim \mu_o$ (vacuum value = $4\pi \cdot 10^{-7}$ Tm/A), one has:

$$n_i^2 = (c_o/c_i)^2 = \varepsilon_i \mu_i / \varepsilon_o \mu_o \sim \varepsilon_i / \varepsilon_o = \varepsilon_r$$

i.e., the relative dielectric permittivity (or dielectric "constant") equals the square of the refractive index, both functions of the radiation frequency. In a more general form, these are both *complex* variables and Maxwell's relation is written as:

$$n^*(\omega)^2 = \varepsilon_r^*(\omega)$$

For common silicate glass, e.g., $\varepsilon_r \sim 5\text{-}10$, but $n_D \sim 1.5$, $n_D^2 \sim 2.25 < \varepsilon_r$. The reason for this apparent failure of Maxwell's relation is the fact that the dielectric constant is normally measured at microwave frequencies (1 MHz), whereas the sodium D line ($\omega = 3.2 \times 10^{15}\ s^{-1}$) lies in the much higher optical frequency domain, where only the electronic polarizability is excited, since only the electrons (but not the ions) are able to follow the oscillations of the electric field of light. In fact, for a material like Ge (which has only electronic polarizability), one has:

$$n_D = 4.0 \qquad \text{and} \qquad \varepsilon_r = 16 = n_D^2$$

in agreement with Maxwell's relation. In fact, from Lorentz-Lorenz relation:

$$[(n^2-1)/(n^2+2)]\ V_M = N_o\ \alpha_t = N_o\ (\alpha_e + \alpha_i + \alpha_o) = \sum_i x_i\ R_{M_i}$$

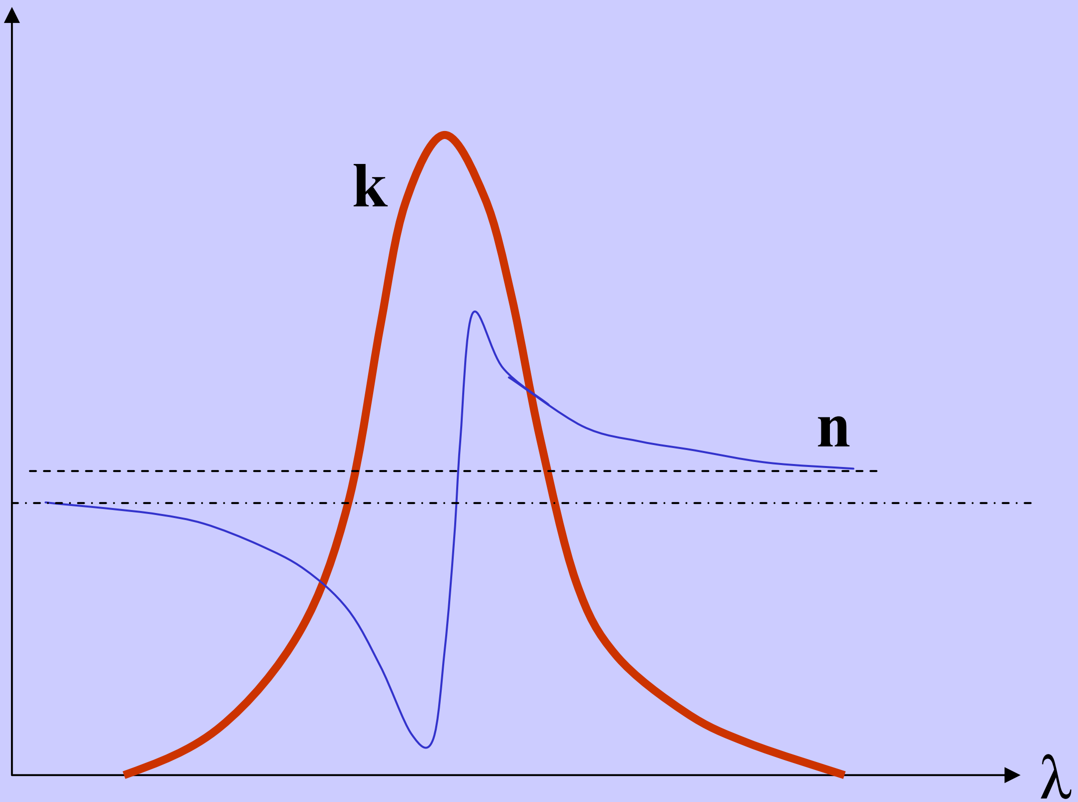
Introducing the *absorption coefficient*, α and the dimentionless *extinction coefficient*, k, one has:

$$n^* = n - i\,k \qquad\qquad \alpha = 4\,\pi\,k / \lambda \quad \text{(in units of } cm^{-1}\text{)}$$

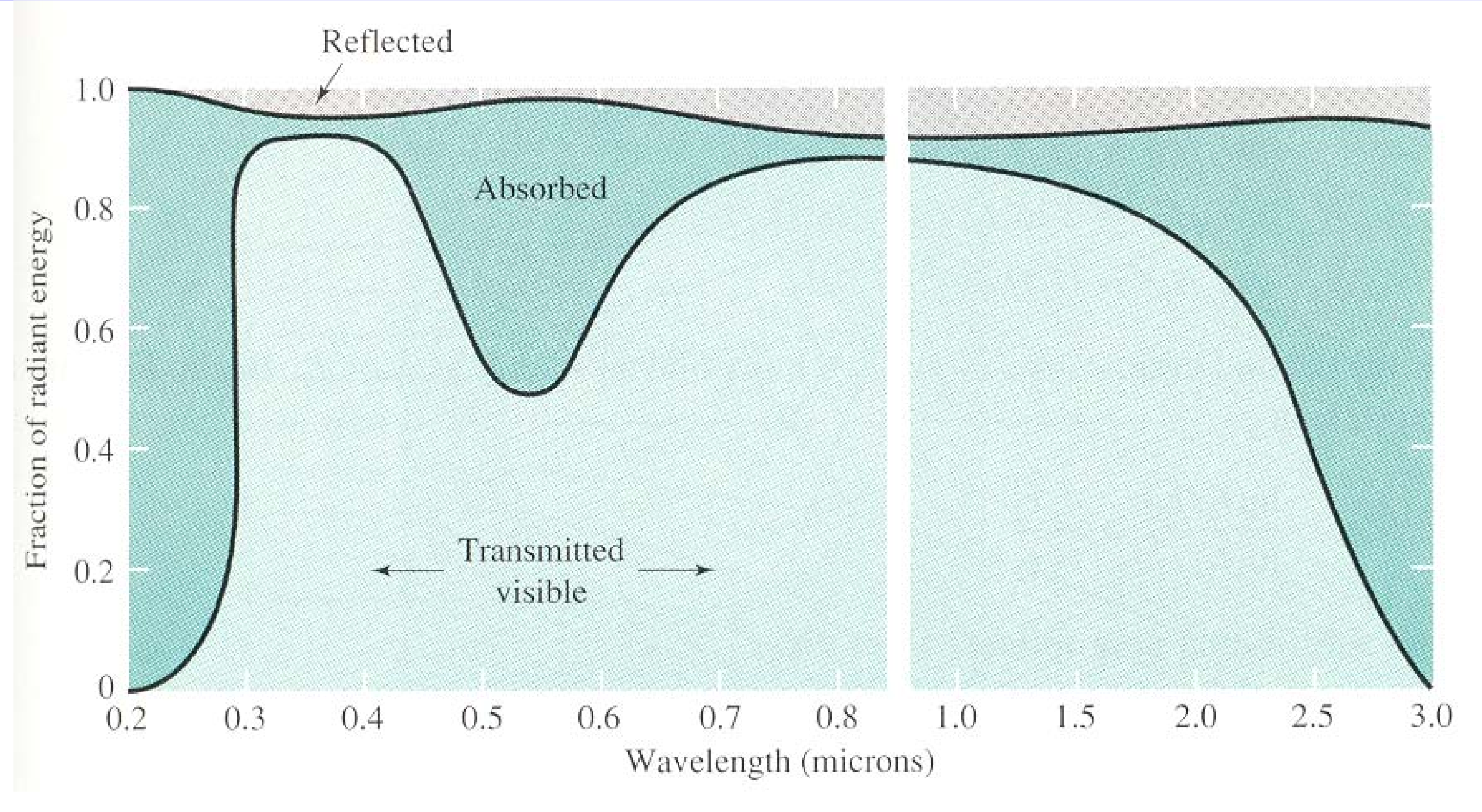
Also:

$$\varepsilon_r^* = \varepsilon_1 - i\,\varepsilon_2 \quad \Rightarrow \quad \varepsilon_1 = n^2 - k^2 \qquad \text{and} \qquad \varepsilon_2 = 2\,n\,k$$

In the vicinity of an absorption band of the glass (where k > 0), the *anomalous dispersion* phenomenon is observed, corresponding to a non-smooth change (including a sudden increase) of n with increasing λ, followed by a normal dispersion region when k becomes zero again. The index becomes higher in this region than before the anomalous dispersion, due to the additional contribution of $\alpha_i$.



Example of the relationship between transmitted, absorbed and reflected light (in the absence of scattering, Kirchoff's law states that T + A + R = 1), for the case of a silicate glass of blue color, due to the presence of $Co^{2+}$ ions.




(Adapted from: *The science and design of engineering materials*, McGraw-Hill, 1999)

For a glass with a "smooth" surface (whose average roughness is < $\lambda/10$), the only component of the reflected light is specular (incidence angle = reflection angle).

For *normal incidence* (incidence angle = $0^o$ off-normal), the Fresnel equation for the reflectivity, R, of the glass surface, is written:

$$R = \left| (n^*-1) / (n^*+1) \right|^2 = [(n-1)^2 + k^2] / [(n+1)^2 + k^2]$$

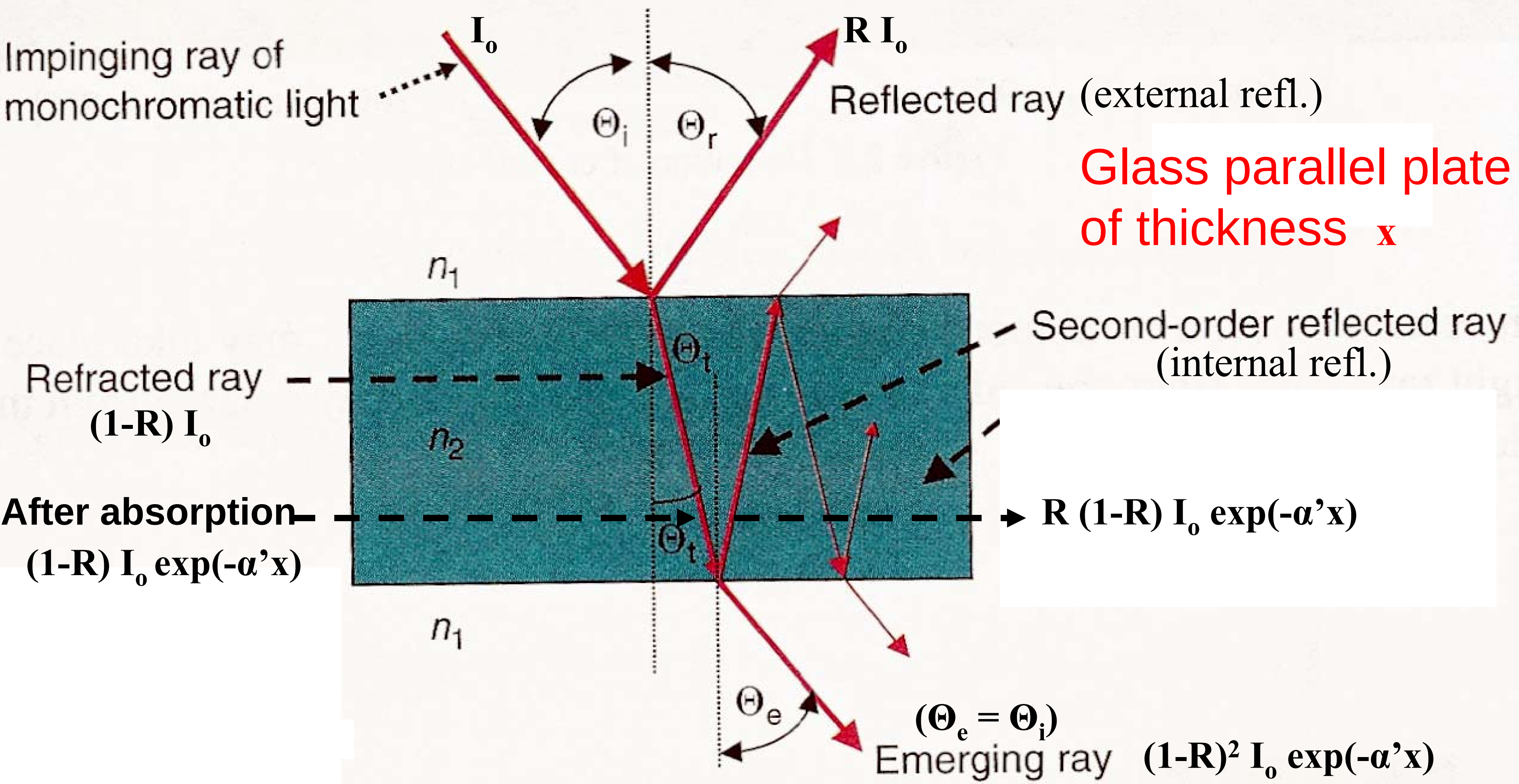
For a glass parallel plate of thickness x and absorption coefficient α' (α' = 2.303 α) , the transmittance is given by Beer's law:

$$T = I/I_o = \exp(-\alpha' x) \qquad \text{or} \qquad A = \log I_o/I = \alpha x$$

If $\alpha' = 1\ cm^{-1}$, then $\alpha = 0.434\ cm^{-1} = 4.34$ dB/cm, sine 1 AU = 10 dB = 10% T.

If reflection is also taken into account, the transmittance can be computed based on the figure below.

(Adapted from: *Introduction to DWDM Technology*, S.V. Kartalopoulos, IEEE Press, 2000)

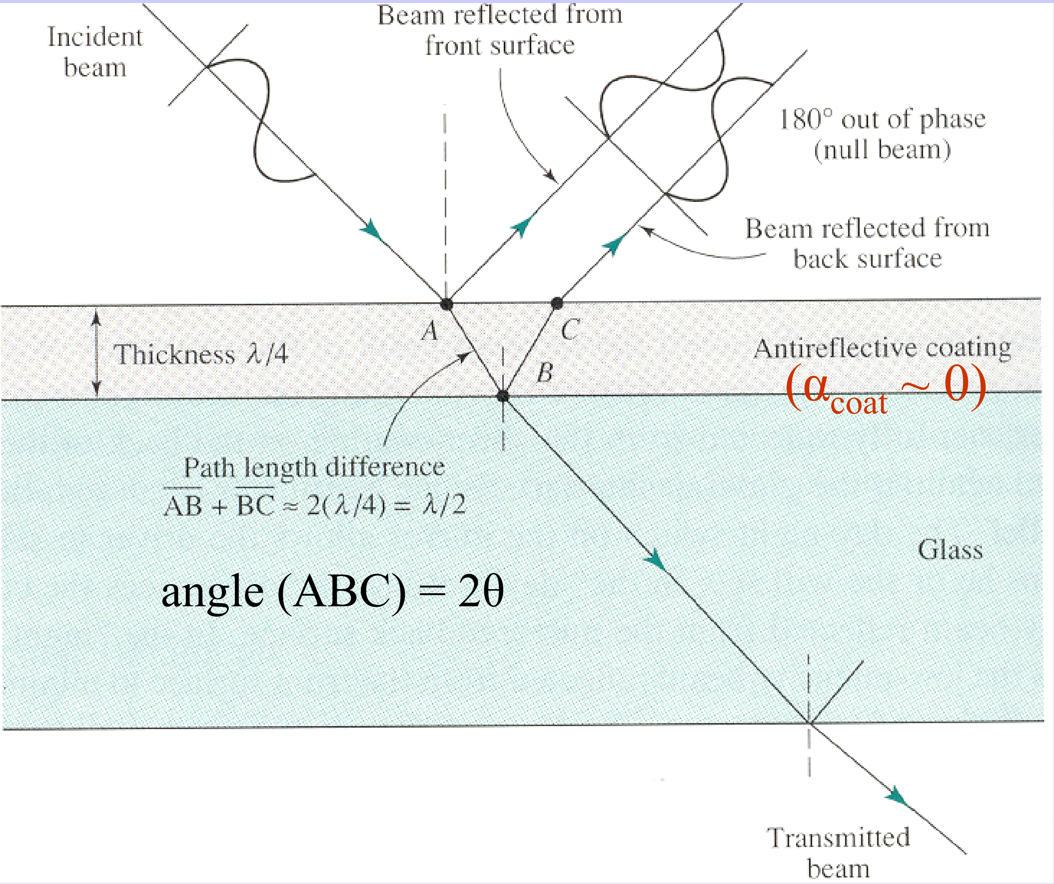



Therefore:

$$T = I / I_o = (1 - R)^2 \exp(-\alpha' x)$$

If α' ~ 0, there are only reflection losses: $T = (1-R)^2 \sim 1-2R = 1-2(n-1)^2/(n+1)^2 \sim 92\ \%$ for common silicate glass with n ~ 1.5 and for $\theta_i$ not higher than ~ 45°.

In applications like ophtalmic lenses, windows or telescopes, it is important to eliminate reflection losses, by means of *anti-reflection coatings*, based on destructive *interference* of the light reflected from the top and bottom surfaces of a film of thickness d and *index* n, *lower than* that of *the glass*. The difference in *optical path length* (the *physical* path length times the refractive index) between the *refracted* and *reflected* rays, taking into account a phase change of π (equivalent to an optical path difference of λ/2) for *external* reflection (light incident on the surface of a more dense medium), is given by:




$$\Delta d = 2nd\cos\theta = (m+1/2)\,\lambda$$

for *destructive* interference (m=0,1,…).

For normal incidence (θ = 0) and m = 0:

$$2nd = \lambda/2 \quad \Leftrightarrow \quad d = \lambda/4n$$

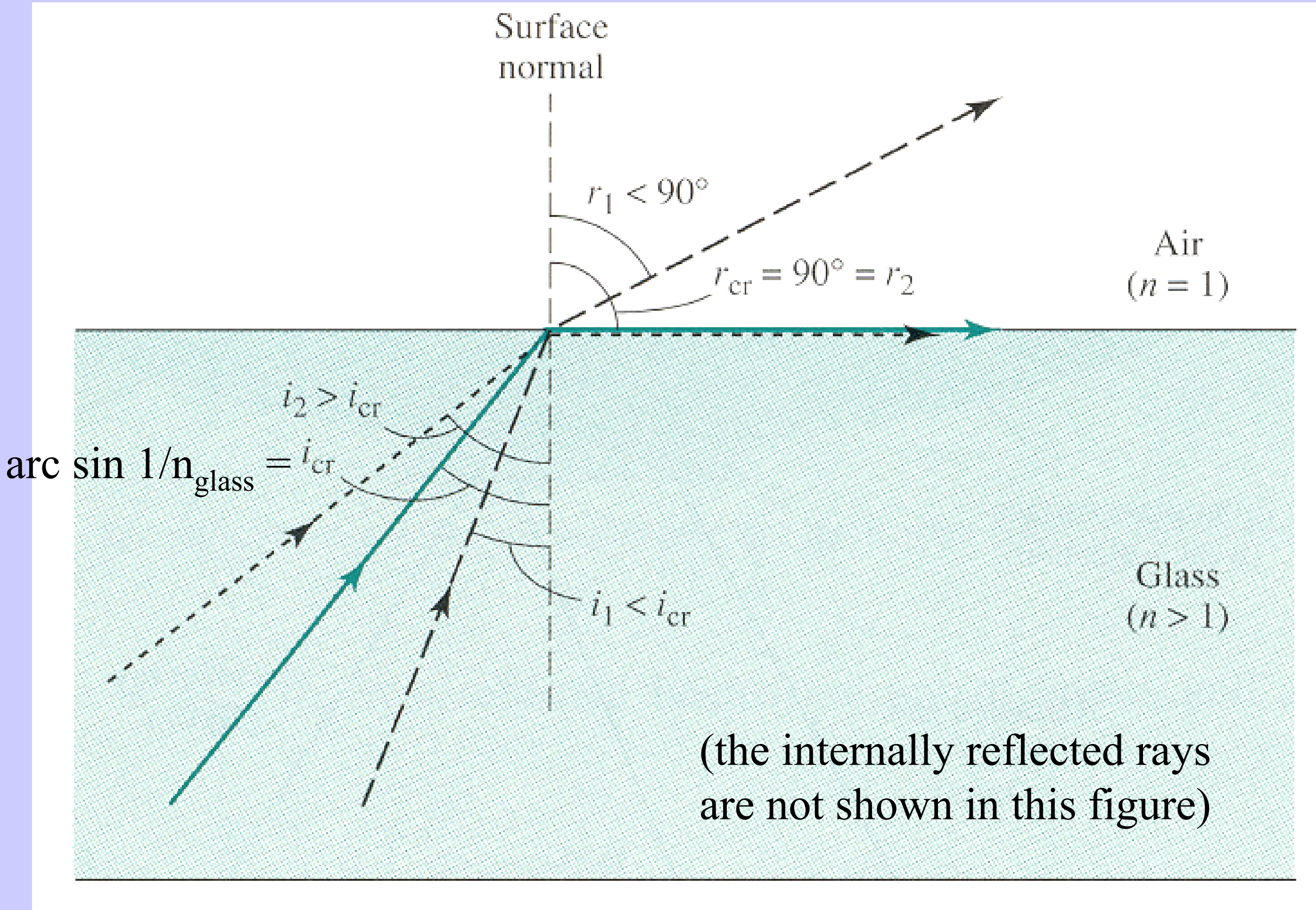
<u>Note</u>: for a film of *index higher than the glass*, *constructive* interference would occur for a film of the same thickness (a *quarter-wave reflective coating)*:

$$\Delta d = 2nd\cos\theta - \lambda/2 = m\lambda \quad (\theta=0 \text{ and } m=0)$$

(Adapted from: *The science and design of engineering materials, J.P. Schaffer et al., McGraw-Hill, 1999)*

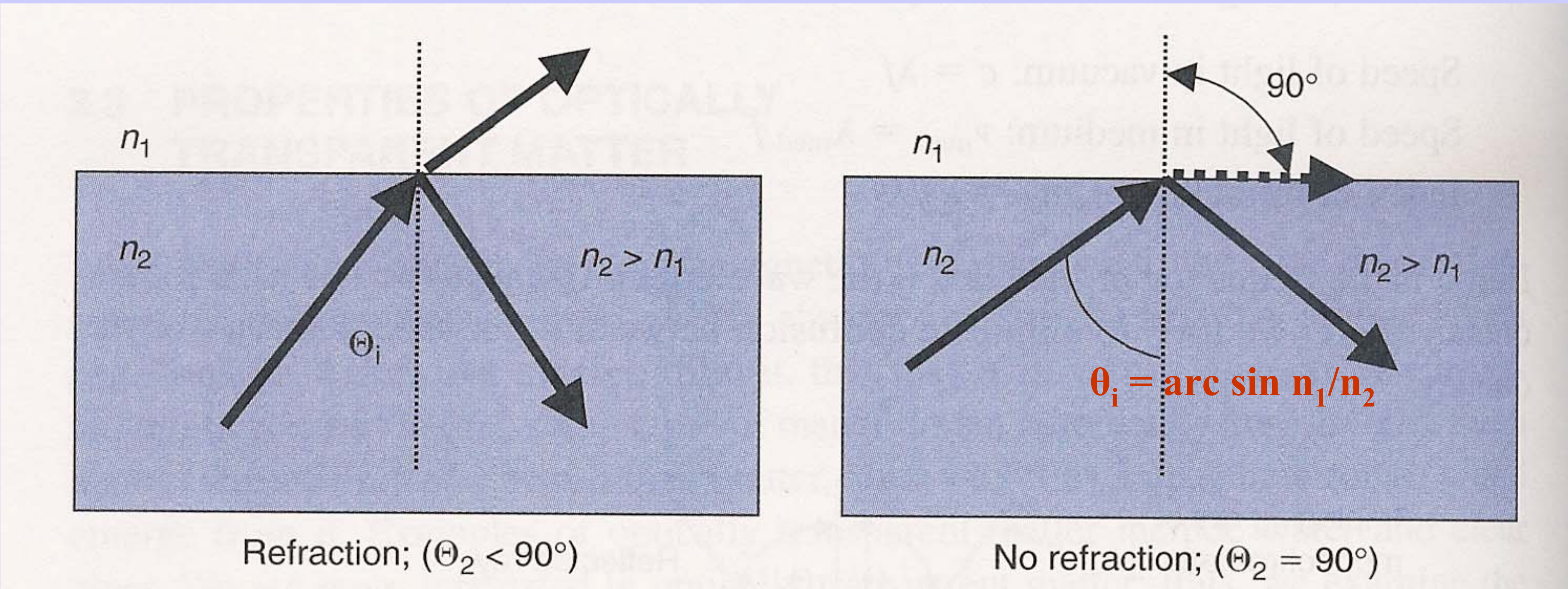
# Total *internal* reflection

When light is incident on the surface of a less dense medium (of lower index) above a critical angle with the surface normal (given as arc sin 1/n, by Snell's law) , total internal reflection occurs, the phenomenon on which guided wave optics and **fiberoptics** in particular are based.




(Adapted from: *The science and design of engineering materials*, J.P. Schaffer et al., McGraw-Hill, 1999)

# Critical angle for total internal reflection




(Adapted from: *Introduction to DWDM Technology*, S.V. Kartalopoulos, IEEE Press, 2000)

The dependence of the glass reflectivity on both **n** and **k** enables the calculation of these **fundamental optical constants** from reflectivity spectra at near-normal incidence, by means of *Kramers-Kronig analysis*.

If r* is the complex amplitude of the reflected light wave, this is a function of the phase difference between the incident and reflected waves (given as a phase angle φ):

$$r^* = |r| \exp(i\varphi)$$

Also, since the reflectivity $R = |r|^2$, one will have, for normal incidence:

$$n = (1-R) / (1+R-2R^{1/2}\cos\varphi) \qquad \text{and} \qquad k = 2R^{1/2}\sin\varphi / (1+R-2R^{1/2}\cos\varphi)$$

If R is spectroscopically measured as a function of the frequency ω, φ is then obtained, for each frequency value $\omega_i$, from the Kramers-Kronig relation:

$$\varphi(\omega_i) = (2\omega_i/\pi) \int_0^{\infty} \{\ln[R(\omega)]^{1/2} - \ln[R(\omega_i)]^{1/2}\}\, d\omega / (\omega_i^2 - \omega^2)$$

The values of n and k as a function of ω allow, e.g., the calculation of the reflection spectra R(ω), also as a function of the incidence angle off-normal, θ, by means of the Fresnel equations for oblique incidence.

For a common silicate glass at a visible frequency ($n \sim 1.5$ and $\alpha \sim 0$), $R(\theta)$ will have the approximate shape shown below, for external reflection of light. Note that the effect of oblique incidence only becomes important for $\theta$ higher than ~ 45$^{o}$ and that external reflection is only total at grazing incidence.