Mechanical Properties of Glass

- Elastic Modulus and Microhardness
  [Chapter 8 – The “Good Book”*]
- Strength and Toughness [Chapter 18]
  - Fracture mechanics tests
  - Fractography
  - Stress Corrosion
  - Fracture Statistics

*A. Varshneya, “Fundamentals of Inorganic Glasses”, Society of Glass Technology (2006)

Questions for homework – Due (to me at jmech@mse.ufl.edu) two weeks after last lecture of this series (Oct. 14) – Due Oct 28, 2008.

(1) Groups of glass specimens having an identical cross-sectional area, square and circular, are being tested for strength under (i) uniaxial stress, (ii) 3-point bend, and (iii) 4-point bend modes. Will there be any difference between the observed strength? Why?

(2) Why does a concentric ring test give a more reliable measure of the strength of glass plates than a 4-point method on beam-shaped specimens of the same plate?

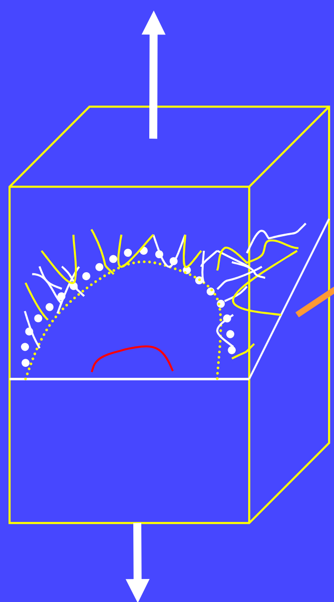
(3) The difference between the breaking strengths of specimens before and after ion exchange strengthening is usually significantly less than the surface compression. Why? What if the surface flaws from the glass were removed (by etching with dilute HF) prior to ion exchanging? [Hint: See D. H. Roach and A. R. Cooper, *J. Am. Ceram. Soc.*, **71** (4), C192 (1988)].

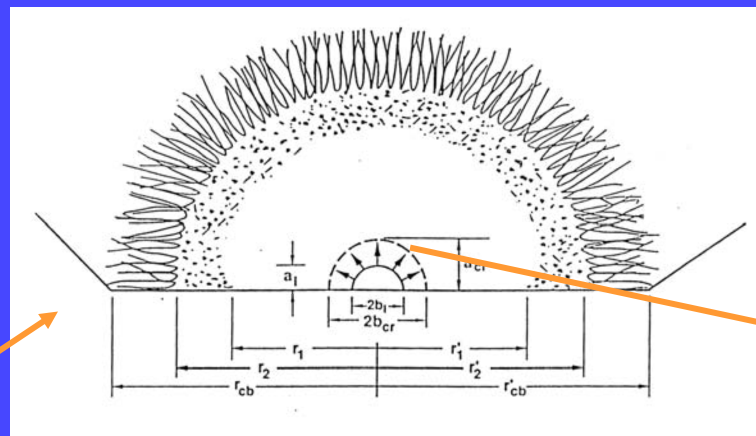
4. In order to test the strength of ceramic, solid cylindrical specimens of length 100 mm and diameter 5 mm are placed in axial tension. The tensile stress, σ, which causes 50% of the specimens to fracture is 120 MPa. For the same material, cylindrical components of 25 mm lengths are required to withstand an axial stress, σ_1, with a survival probability of 99%. Given that m = 5 for this material, determine σ_1.

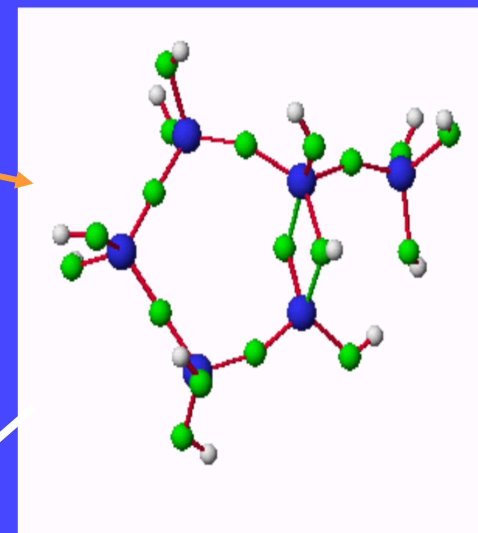
5. As a materials engineer you are required to design a glass window for a vacuum chamber. The opening can be adjusted for a circular disc of radius R and thickness t. It is freely supported in a rubber seal around its periphery and subjected to a uniform pressure difference Δp = 0.1 MPa. The window is a critical component and requires a failure probability of 10^{-6}. The design life of the component is 1000 hours. The modulus of rupture tests of the glass discs to be used resulted in a mean strength of 300 MPa in a short term (60 second) bending test. What are the permissible dimensions for this window? [Assume Poisson's ratio is 0.25, the Weibull's modulus is 5 and the stress corrosion susceptibility parameter is 5. Assume the elastic modulus is 70 GPa and K_{IC} = 0.75 MPa $m^{1/2}$. Further, assume the maximum stress in the plate is $\sigma_{max} \sim \Delta p\ R^2 / t^2$; Show all work].

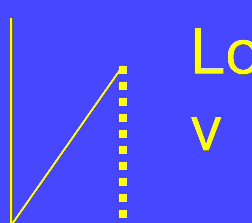
6. You are offered an opportunity to earn $10 million by simply hanging on a rope for only one minute. The rope is attached to a glass sheet (300 cm long by 10 cm wide and 0.127 cm thick). Complicating the situation is the fact that: (a) the glass sheet contains a central crack with total length of 1.62 cm that is oriented parallel to the ground.; (b) the rope is suspended 3 m above a pit of poisonous snakes. The fracture toughness of the glass is 0.75 MPa $m^{1/2}$. Would you try for the prize? Explain why by showing the calculation that demonstrates you could receive the prize or would die trying.

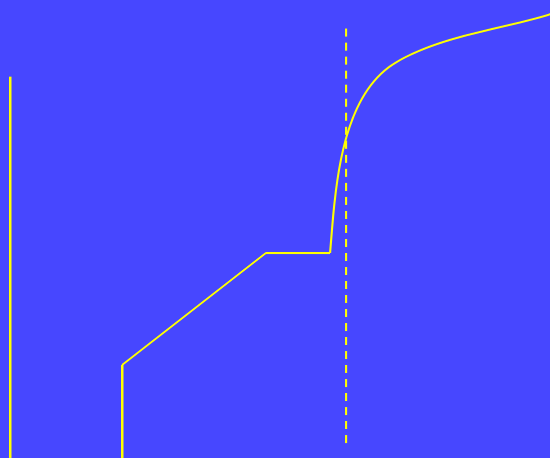
Bond Breaking Leads to Characteristic Features

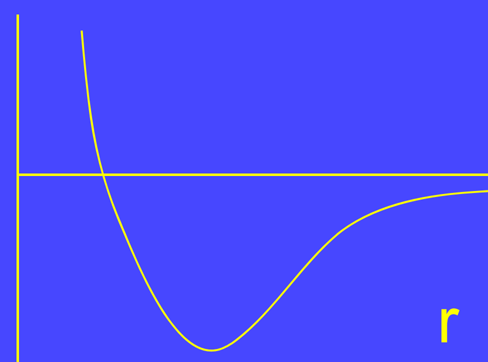








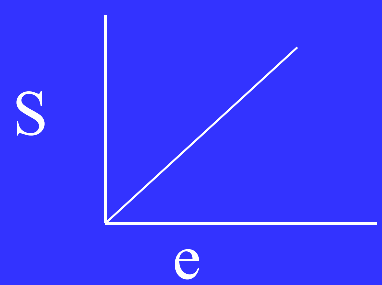


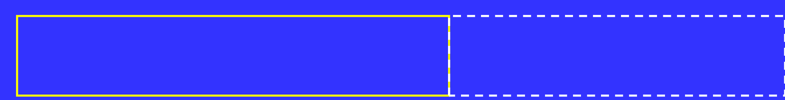


$$\text{Log } K = \text{Log } (Y\sigma\, c^{1/2})$$

There Are Several Important Properties in Mechanical Behavior:

Elastic Modulus – Governs Deflection



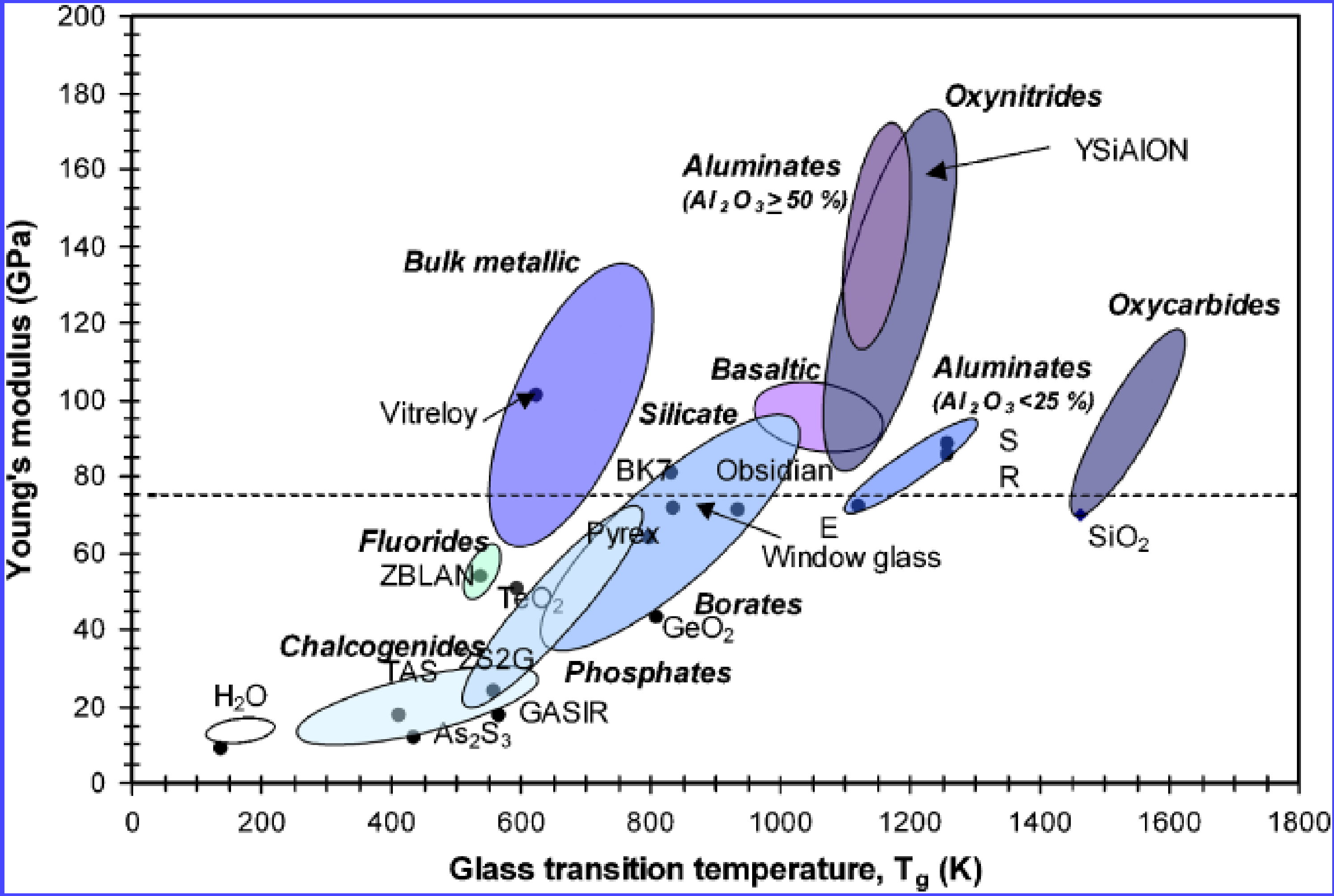


Hardness Measures Surface Properties

Strength – Governs Load Bearing Capacity

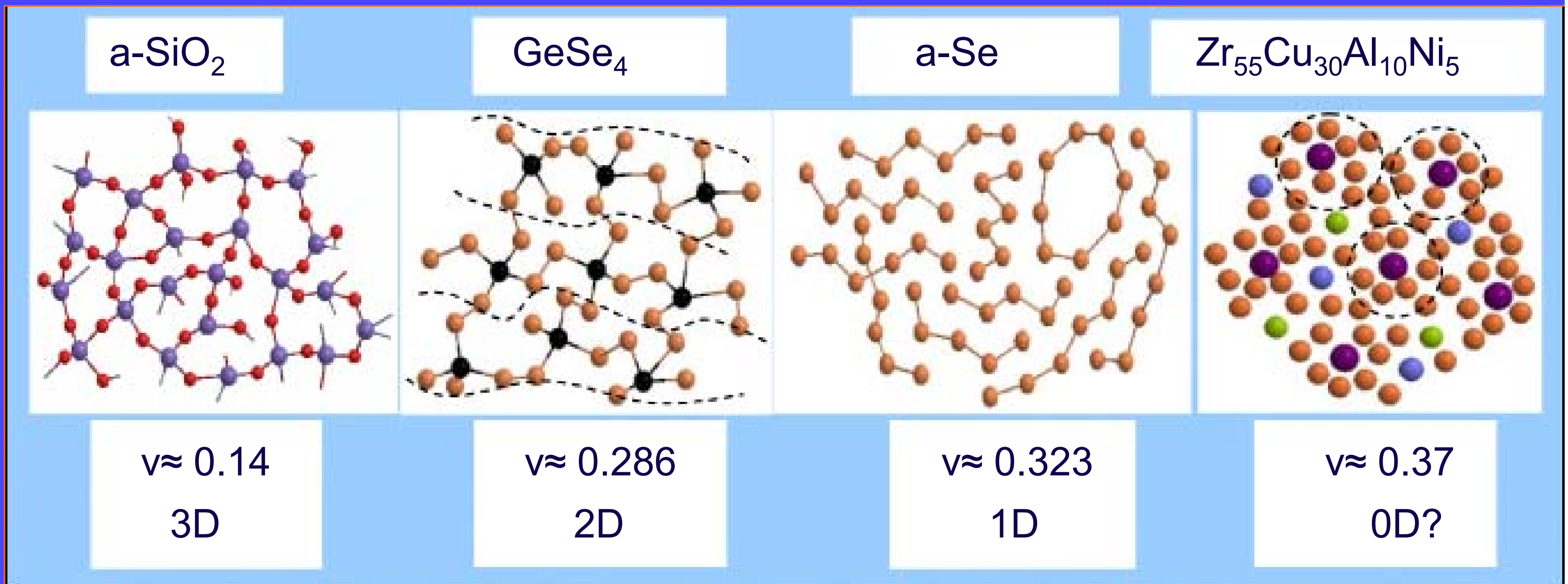
Toughness – Governs Crack Propagation

E & Tg are related within a composition class



"Elastic Properties and Short-to Medium-Range Order in Glasses"
Tanguy Rouxel, J. Am. Ceram. Soc., 90 [10] 3019–3039 (2007)

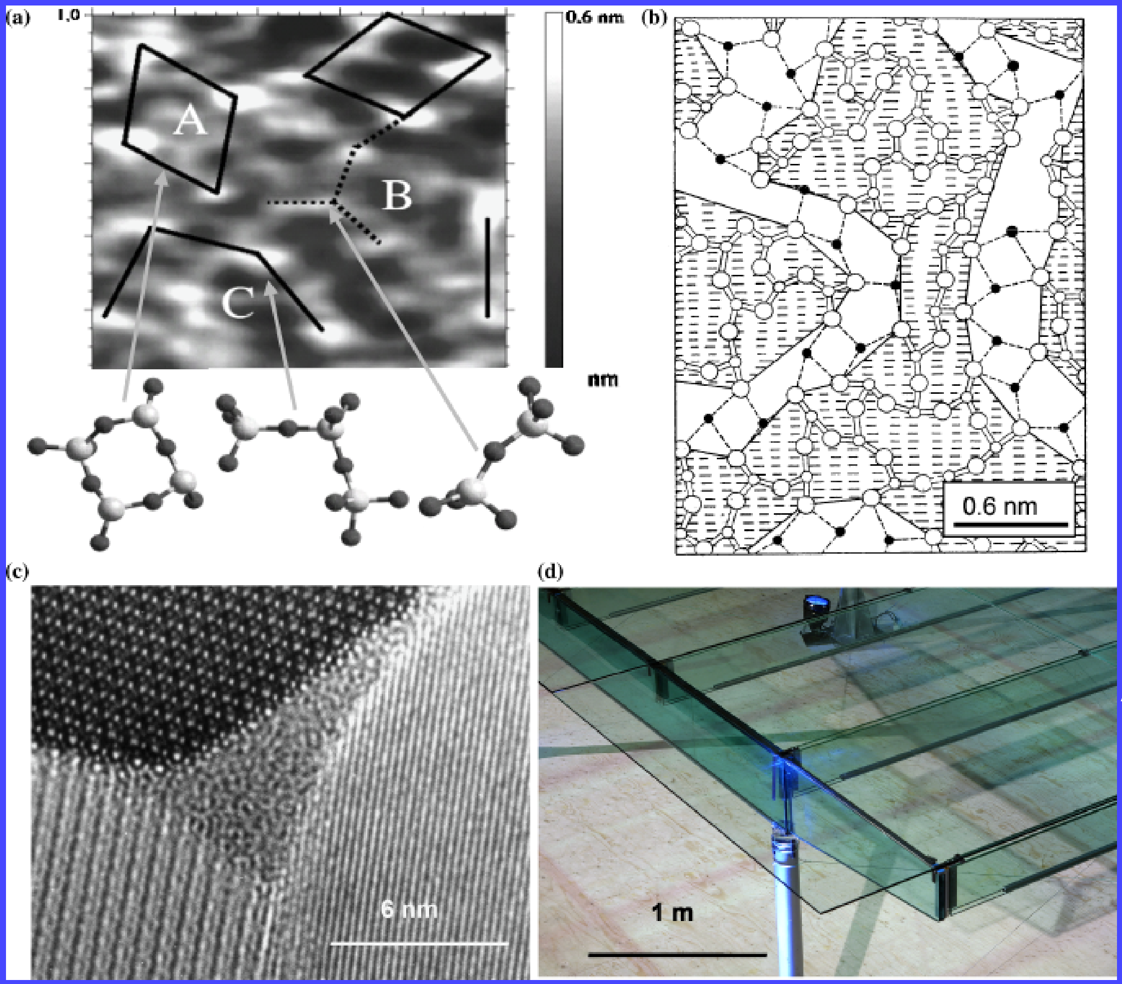
Poisson's ratio (ν) correlates with the atomic packing density (Cg) and with the glass network dimensionality



"Elastic Properties and Short-to Medium-Range Order in Glasses"
Tanguy Rouxel, J. Am. Ceram. Soc., 90 [10] 3019–3039 (2007)

Structure may be viewed on many length scales

AFM
Structural units and arrangements e.g., SiO_4
[JNCS 281,221(2001)]



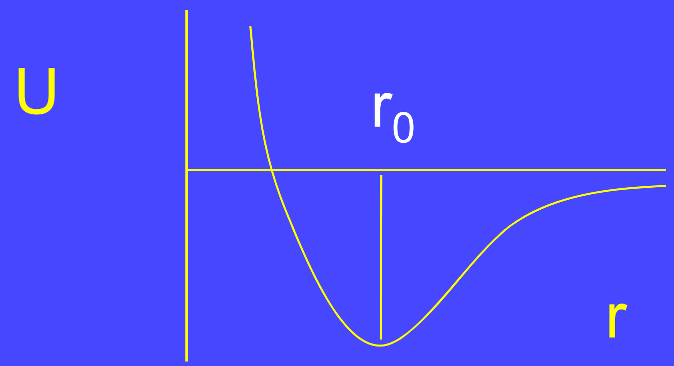
Alkali rich channels [Greaves JNCS 71,203 (1985)]

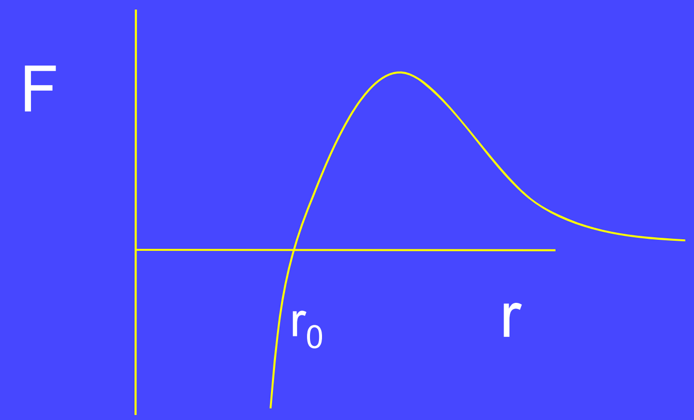
Glassy pocket in Si_3N_4
Acta Met Mater 41,3203(1993)

ALL TRANSPARENT PAVILION
[WWW.GLASS.BK.TUDELFT.NL]

“Elastic Properties and Short-to Medium-Range Order in Glasses”
Tanguy Rouxel, J. Am. Ceram. Soc., 90 [10] 3019–3039 (2007)

Elastic Modulus Is Related To The Strength of Nearest Neighbor Bonds





Force = F = - dU/dr

Stiffness = S_0 = $(dU^2/dr^2)_{r = r0}$

Elastic Modulus = E = S / r_0

Theoretical Strength Can Be Estimated From Potential Energy Curve

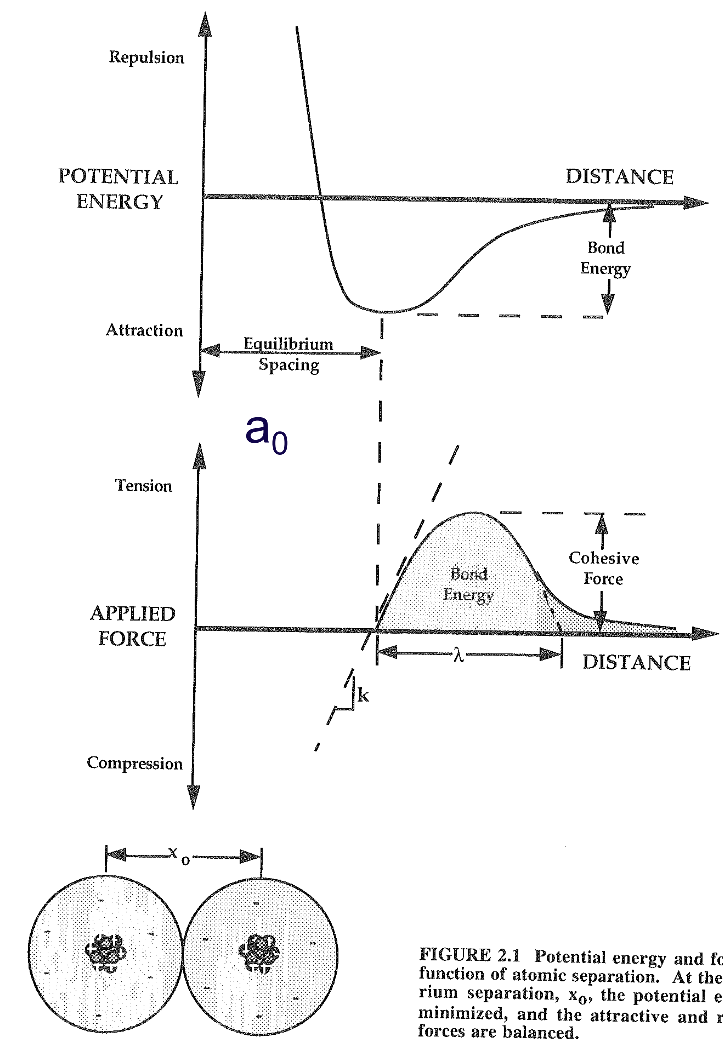


FIGURE 2.1 Potential energy and force as a function of atomic separation. At the equilibrium separation, x_0, the potential energy is minimized, and the attractive and repelling forces are balanced.

$$\sigma_m = \lambda E / \pi a_0$$

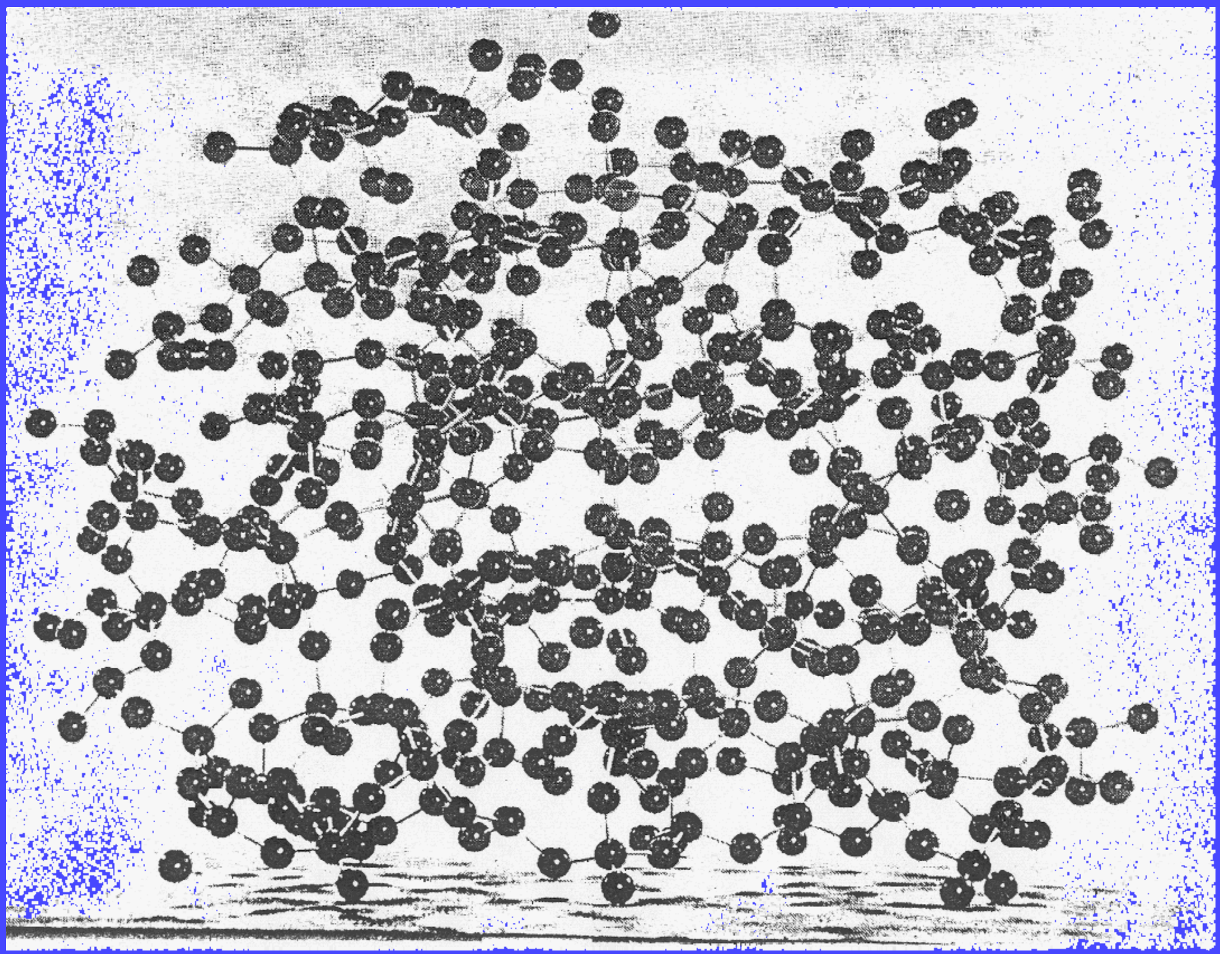
$$2\gamma_f = \sigma_m \sin(\pi x/\lambda)\, dx$$
$$= \lambda\sigma_m / (\pi)$$

$$\sigma_m = [\gamma_f E / a_0]^{1/2}$$

If $E = 70$ GPa, $\gamma_f = 3.5\ \mathrm{J/m^2}$ and $a_0 = 0.2$nm, then

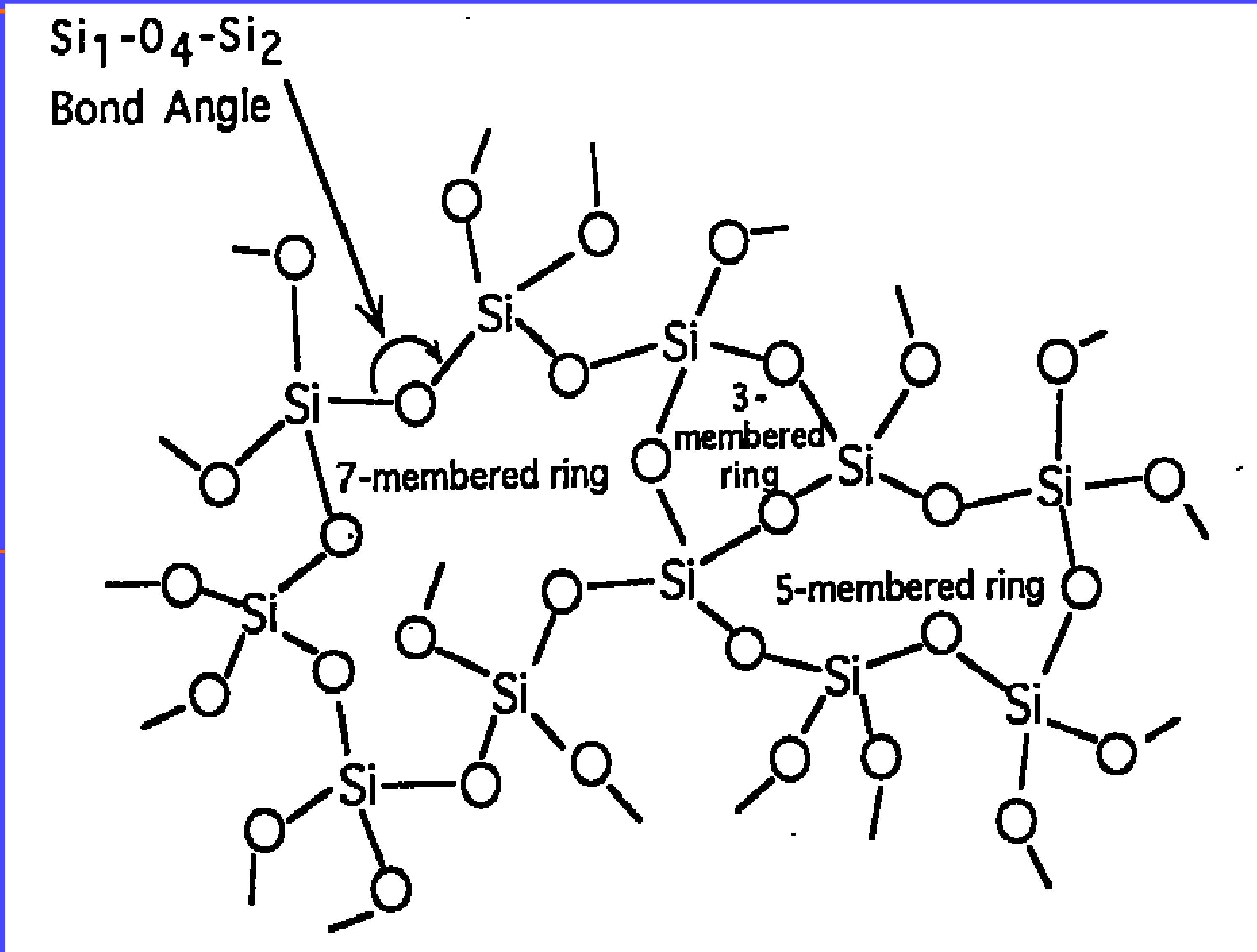
$\sigma_m = 35$ GPa !

Bell & Dean Model Used for MO Calculations



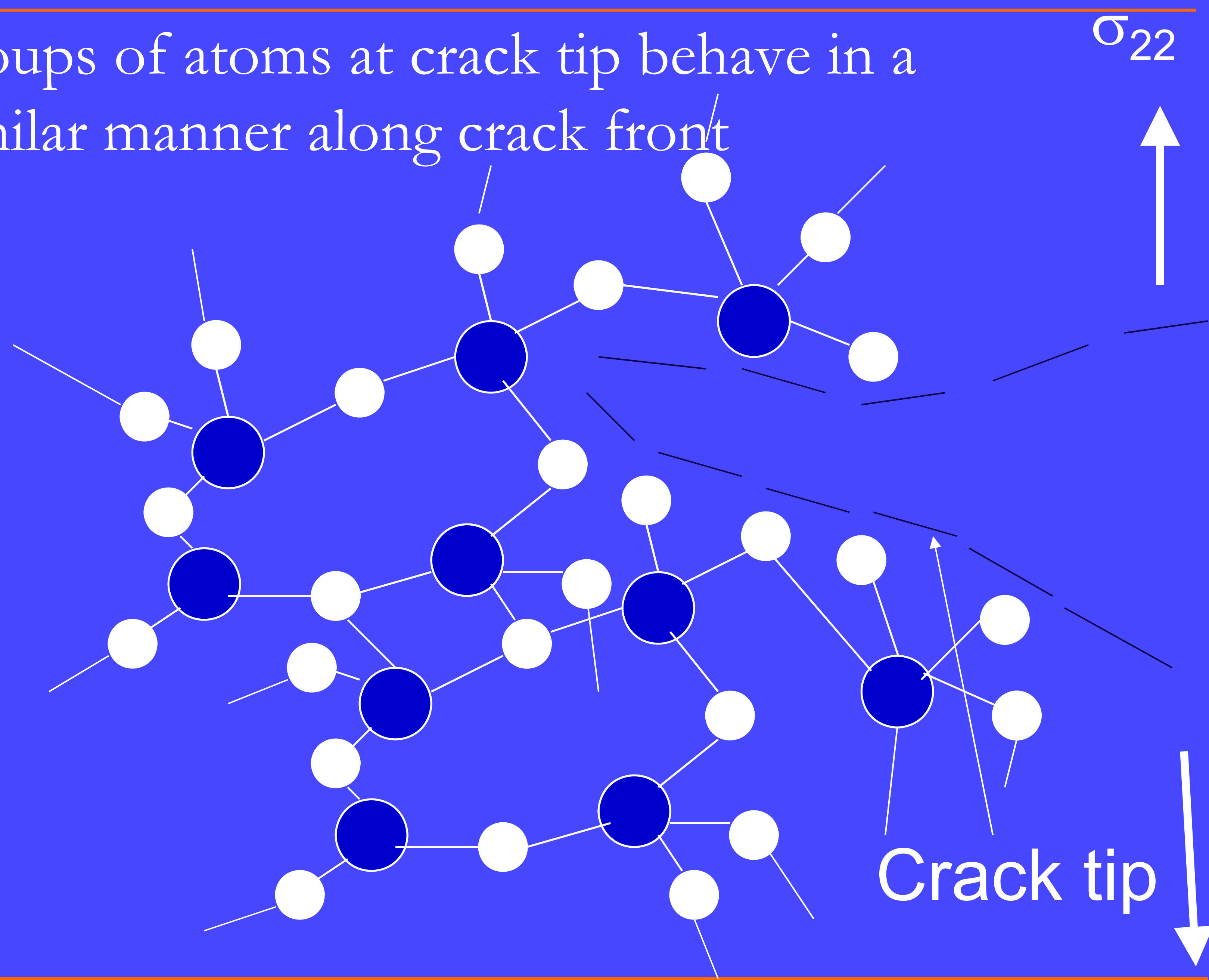
cf. Varsheneya, Fundamentals of Inorganic Glasses

(After Bell and Dean, Nature 212, 1354 [1966])

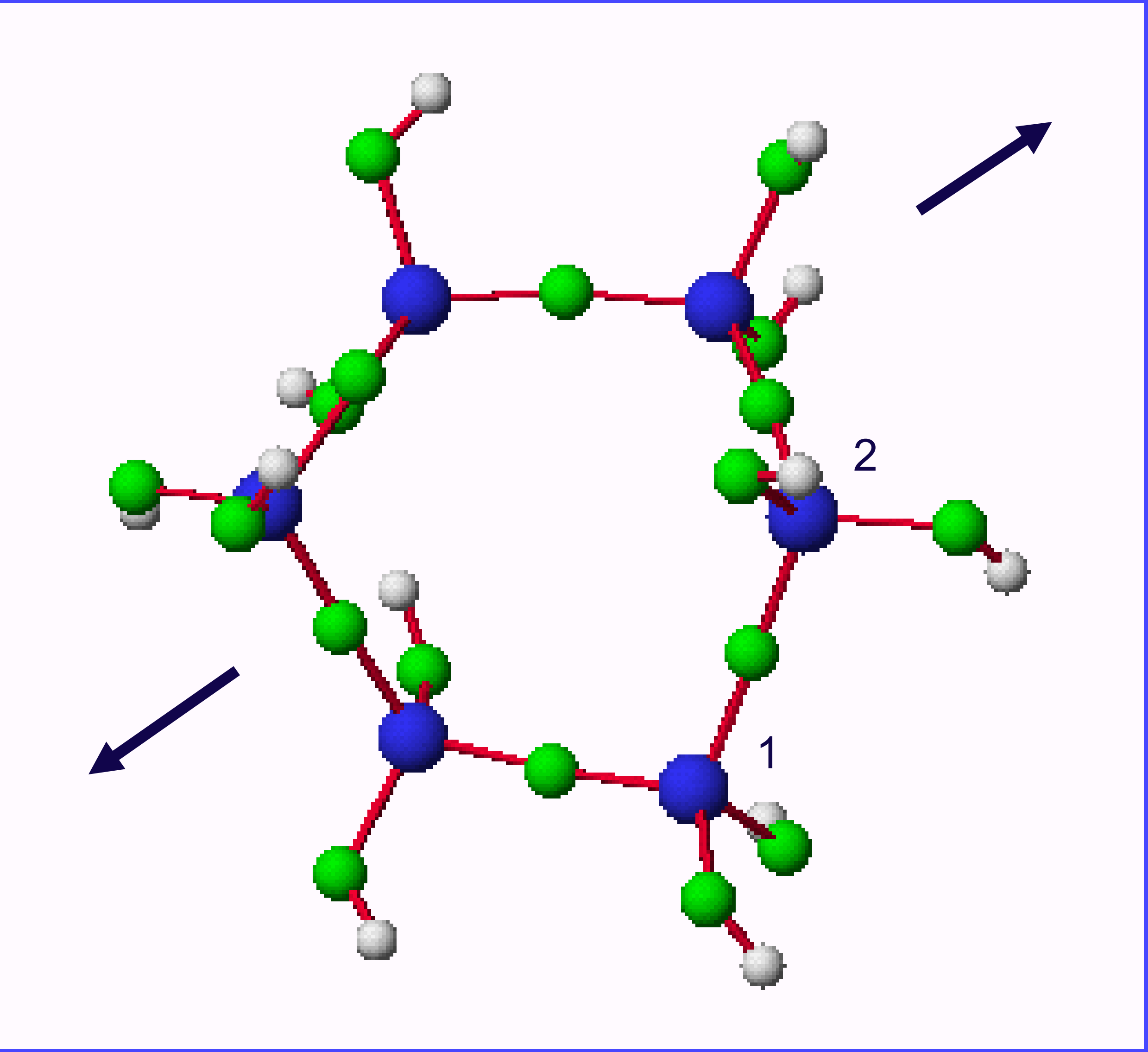


cf. Varsheneya, Fundamentals of Inorganic Glasses

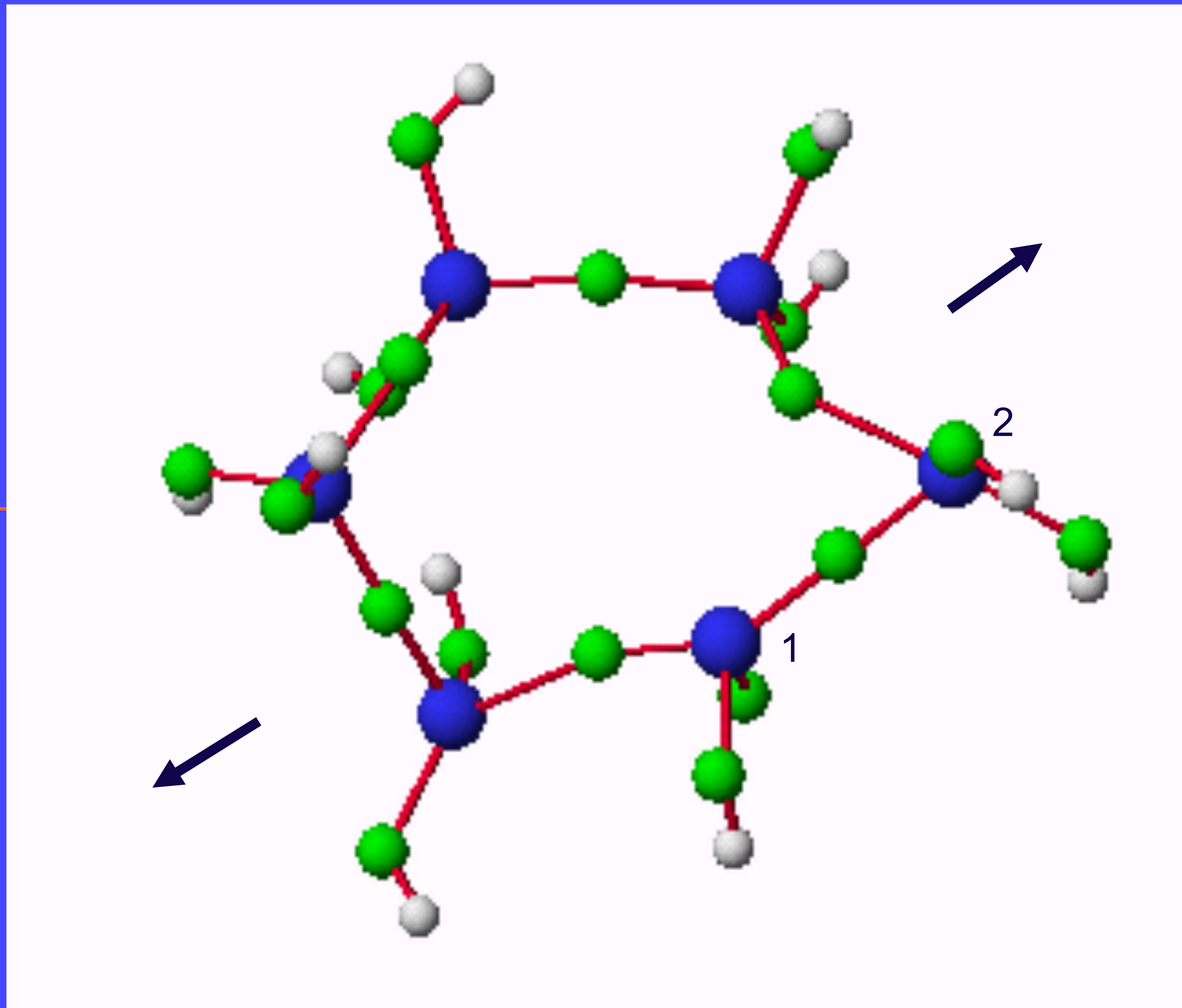
Groups of atoms at crack tip behave in a similar manner along crack front



MO Simulates Bond Breaking At The Crack Tip



Simulated SiO_2 Δ displ.= 1 Å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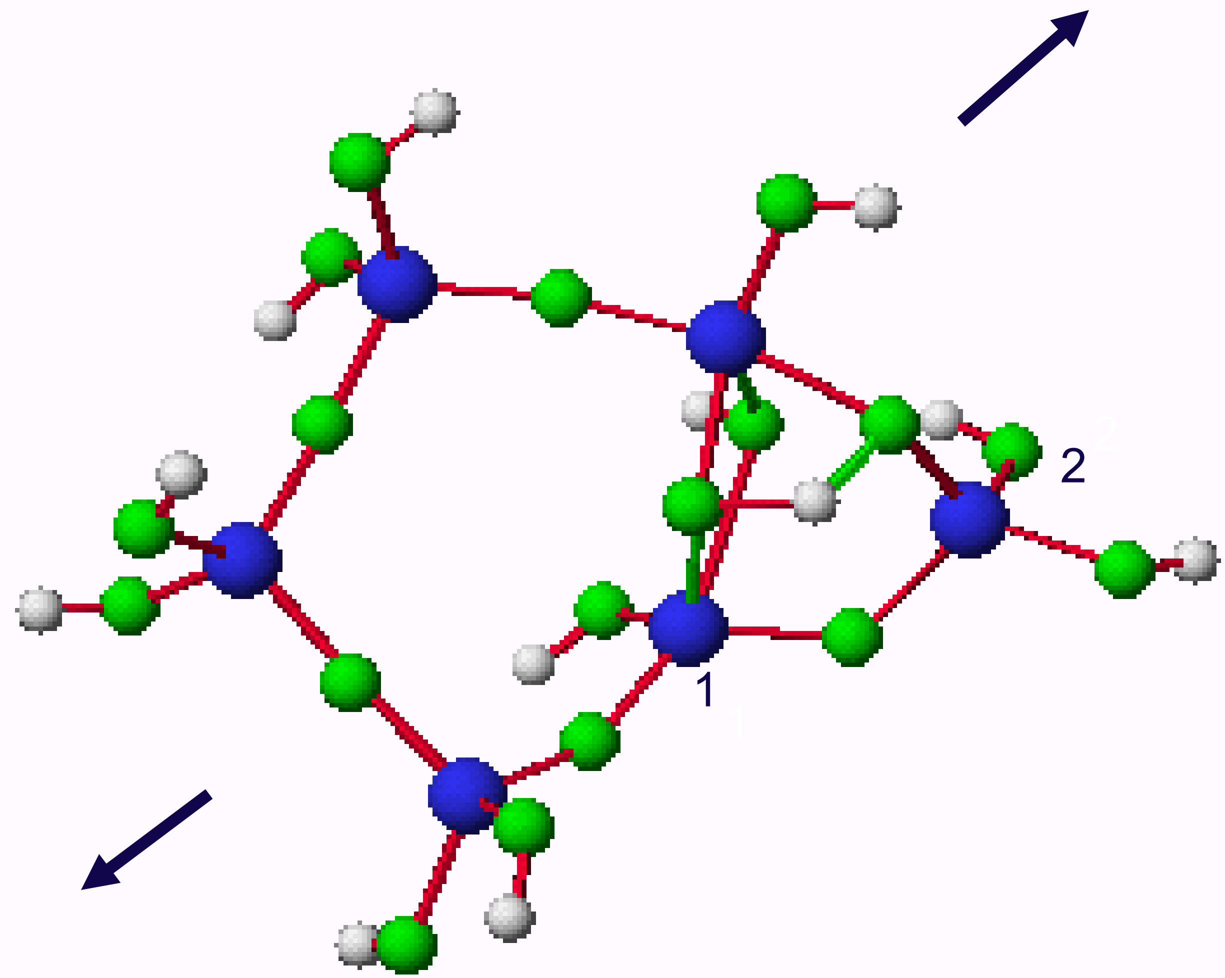
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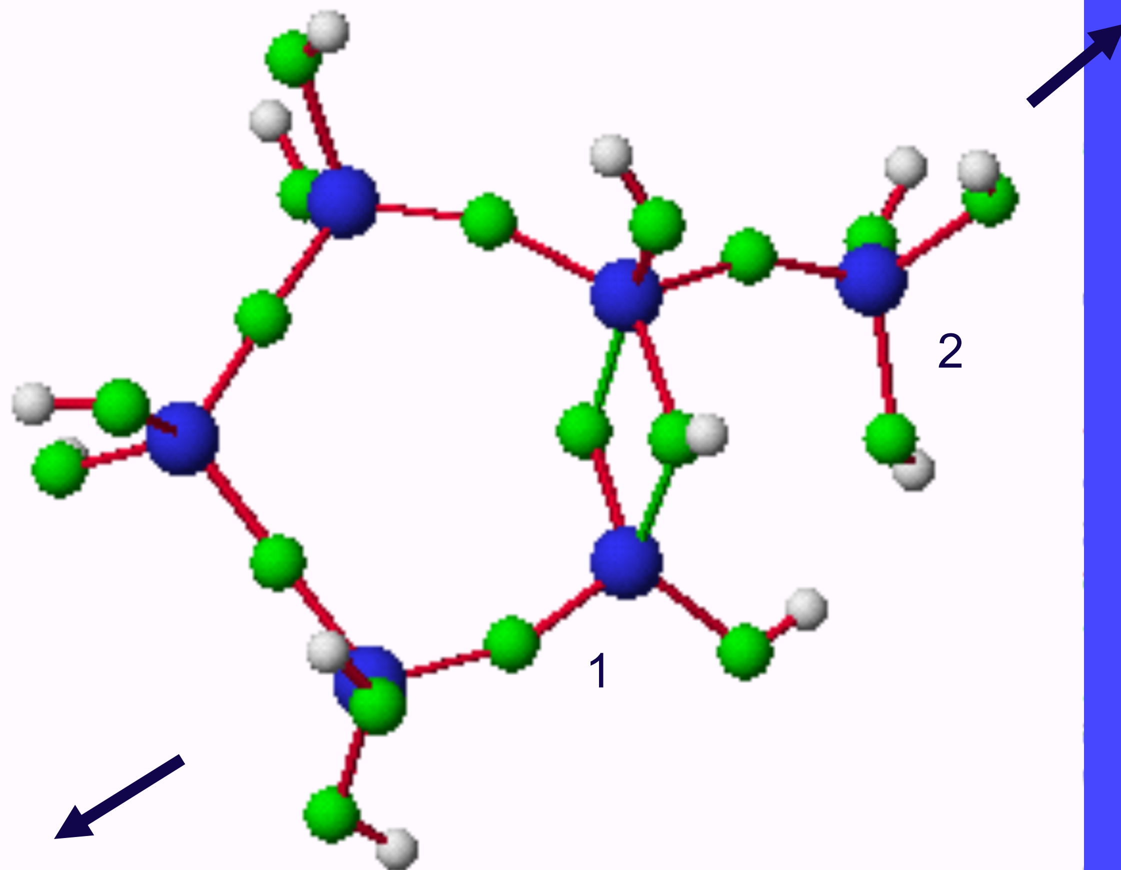
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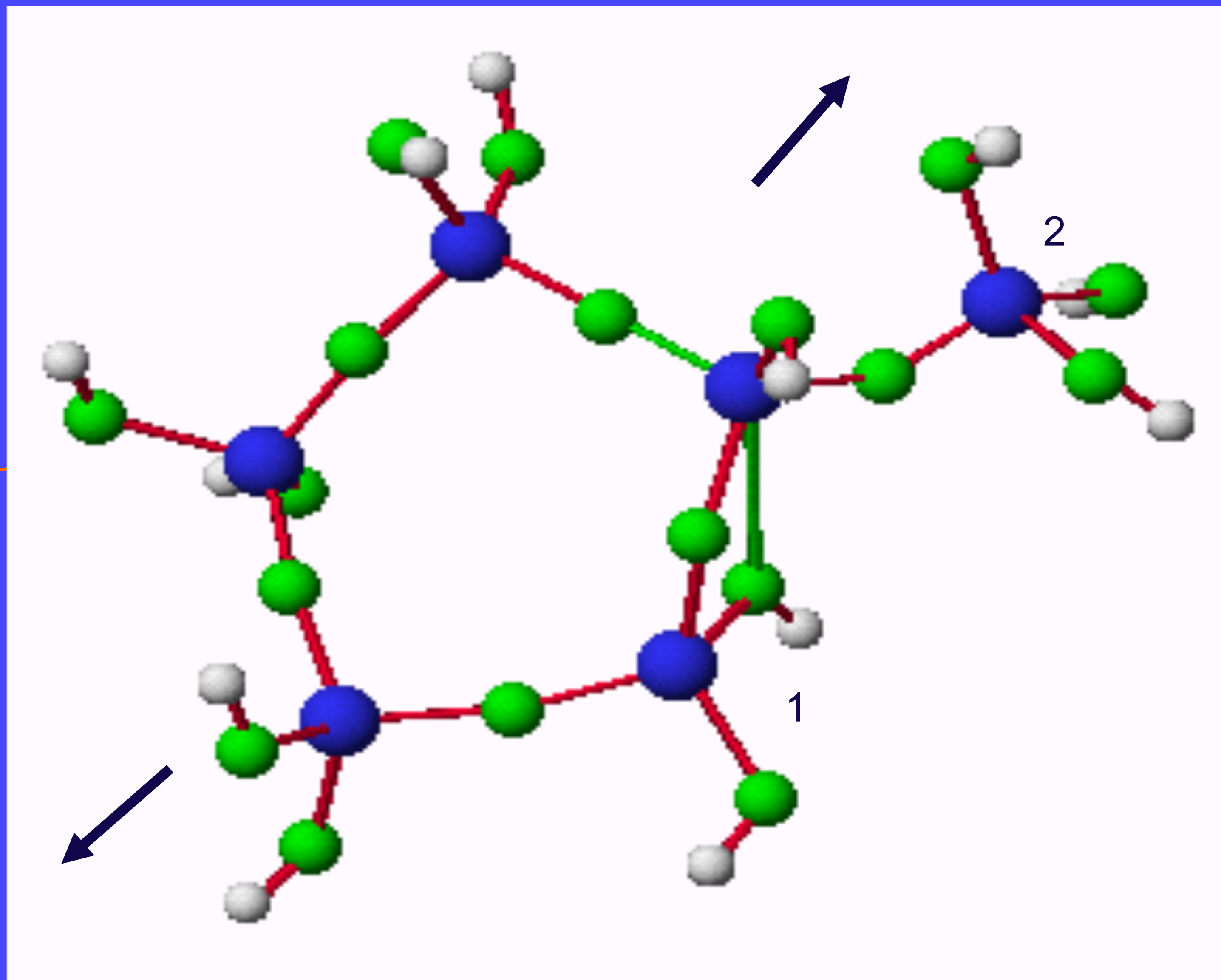
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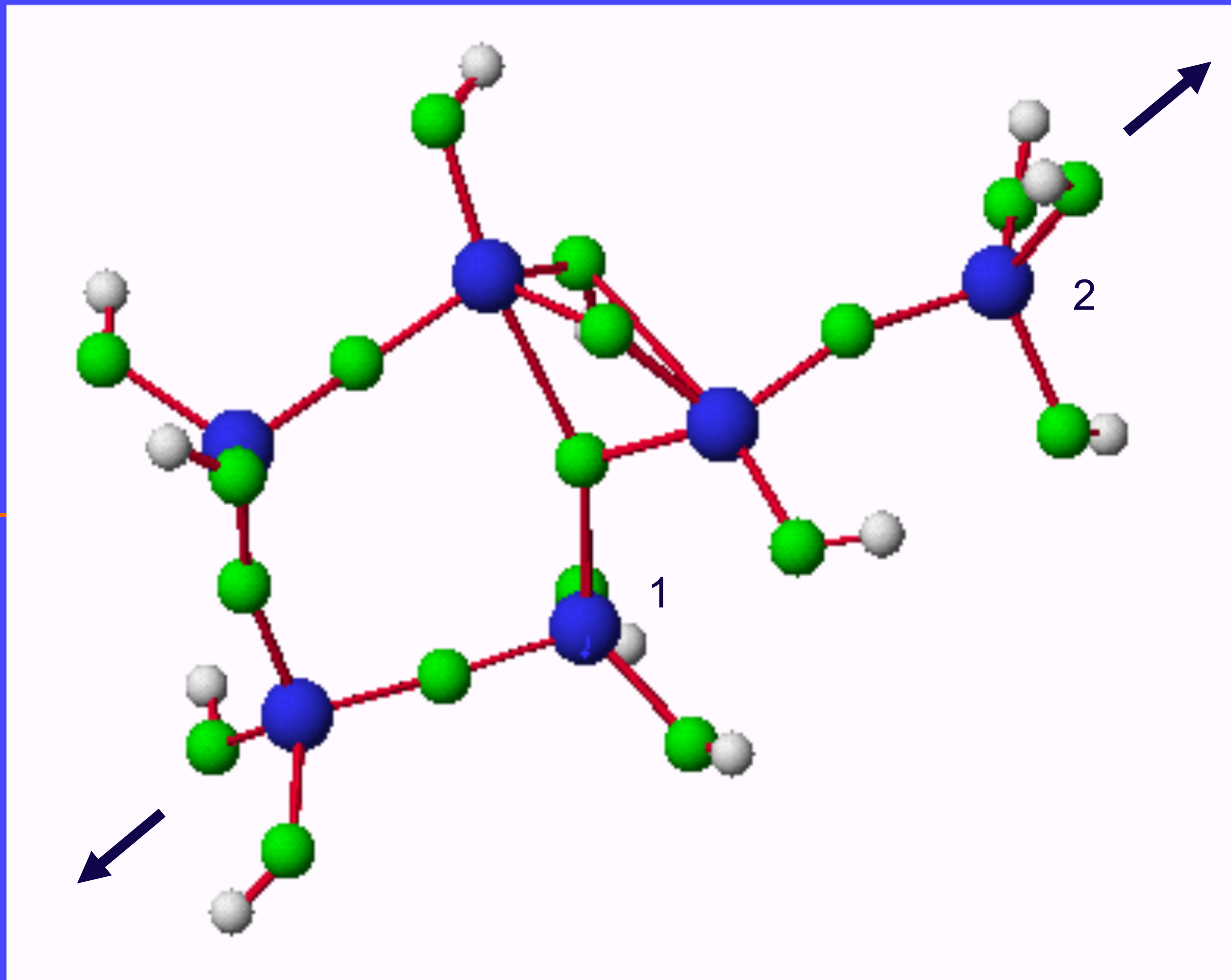
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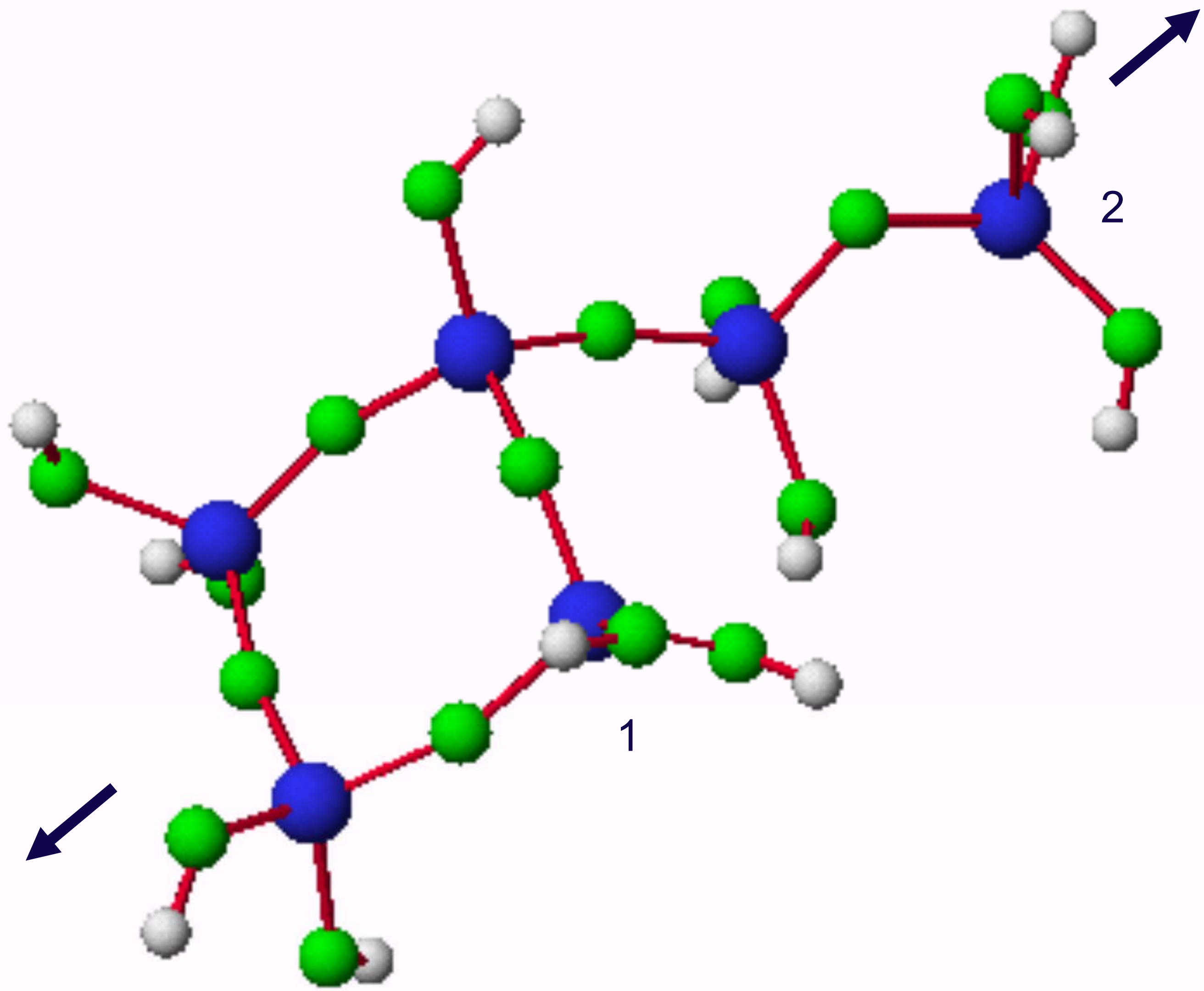
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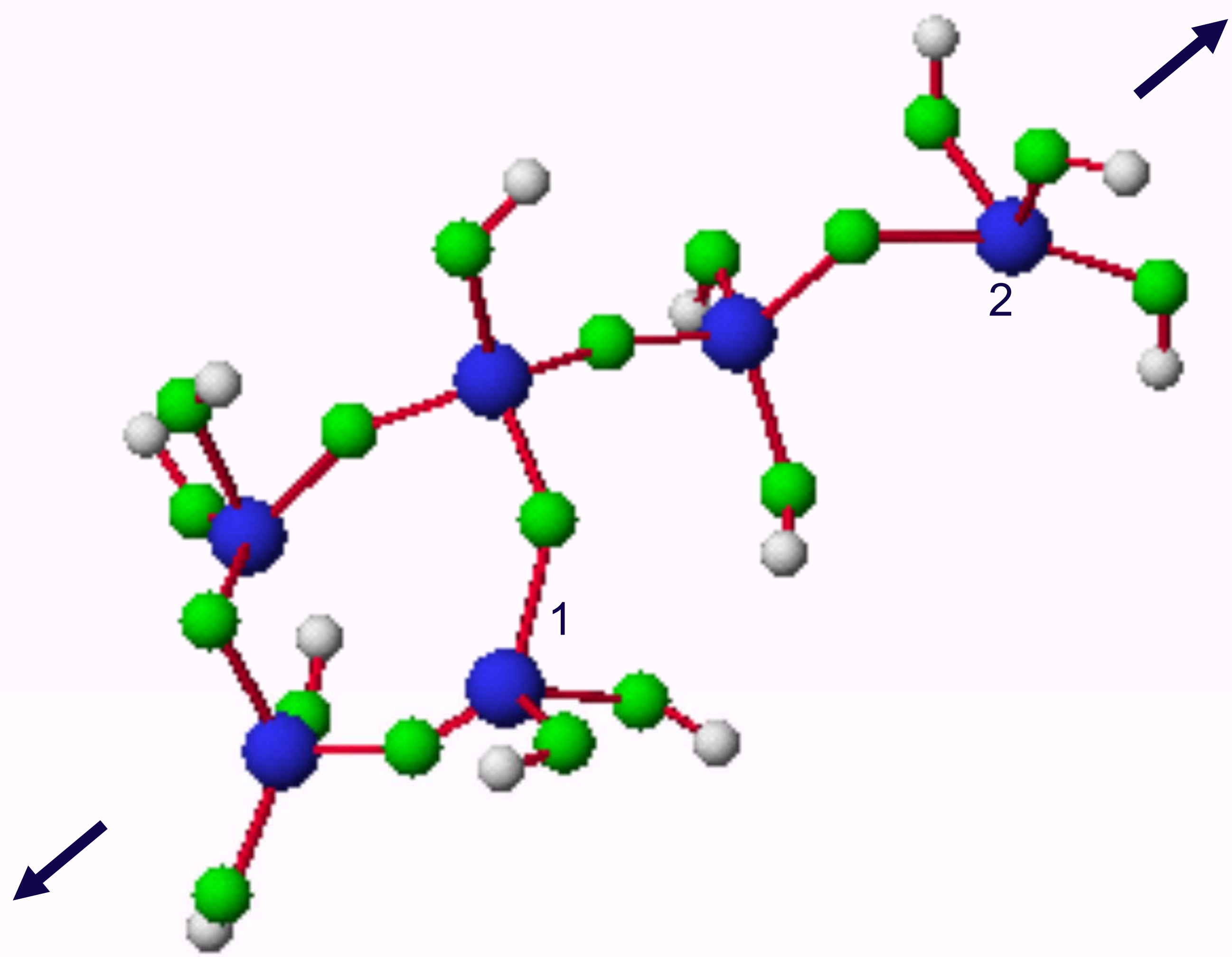
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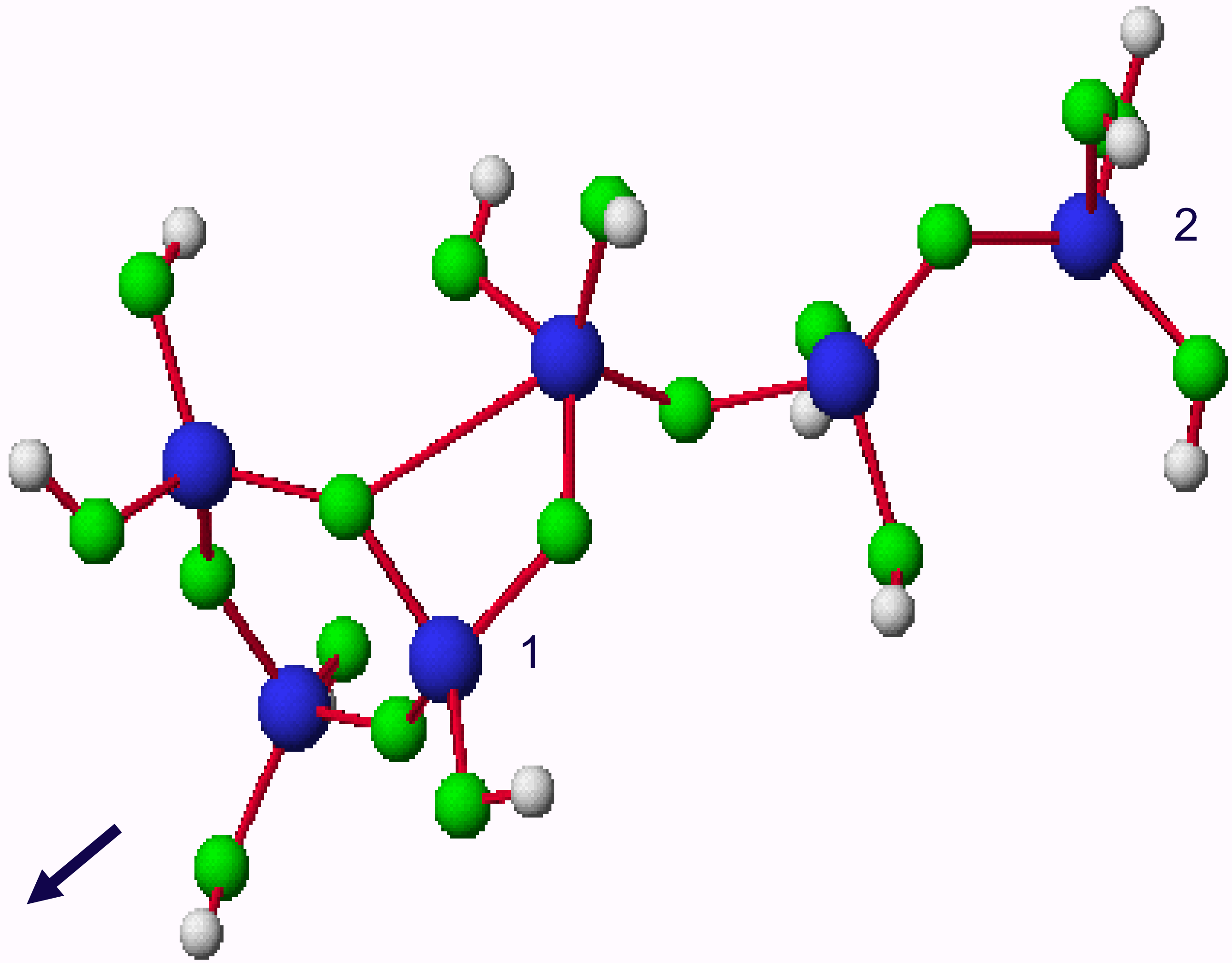
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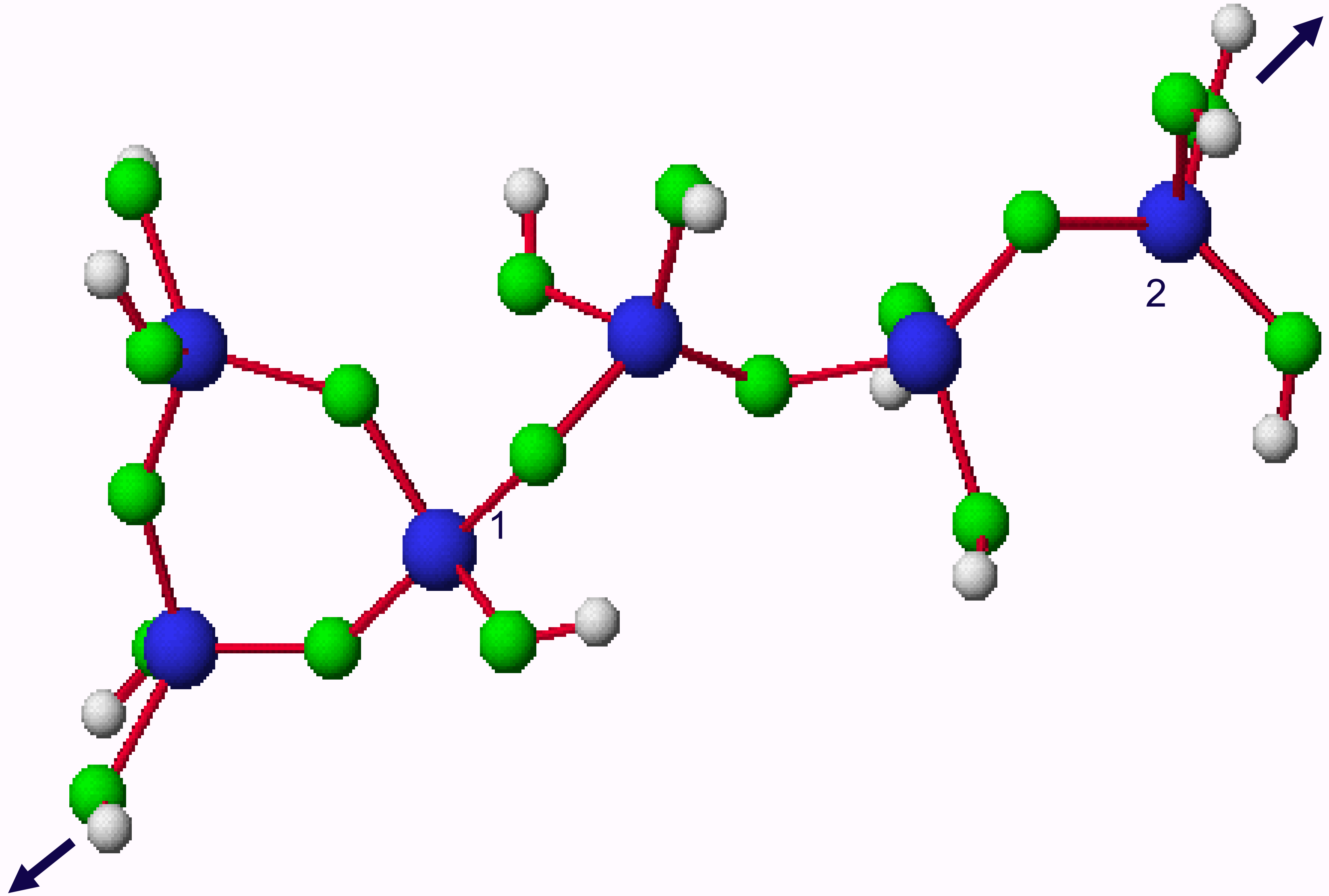
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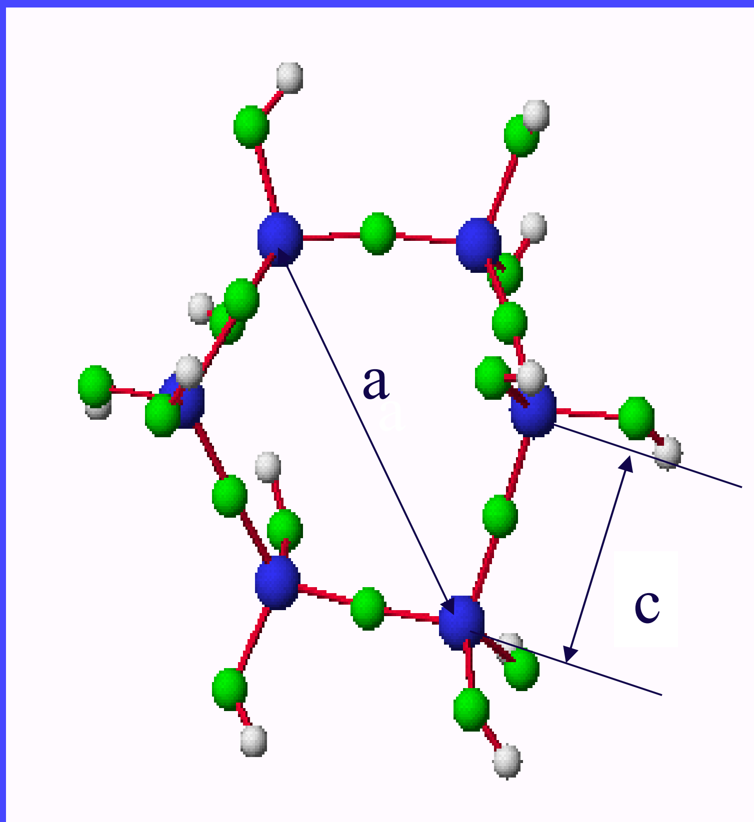
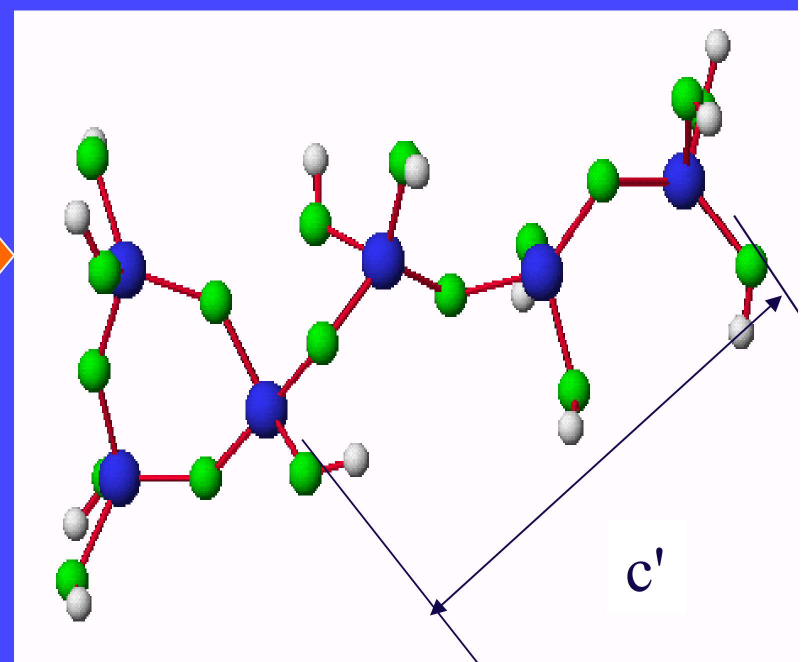
Strain Can Be Calculated By Modeling



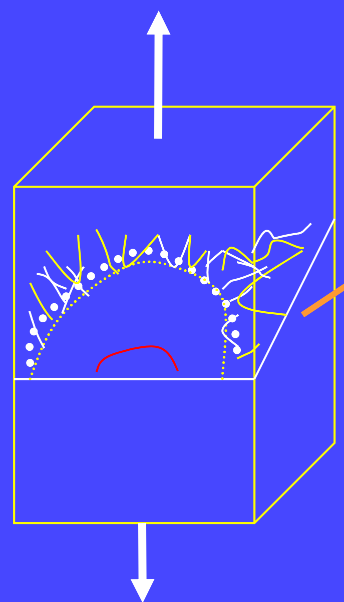
$$a_0 = a / \varepsilon$$

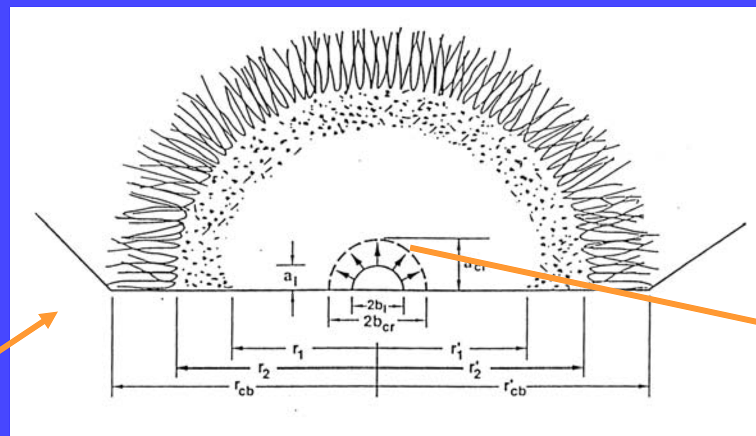
$$= c\, a / c'-c$$

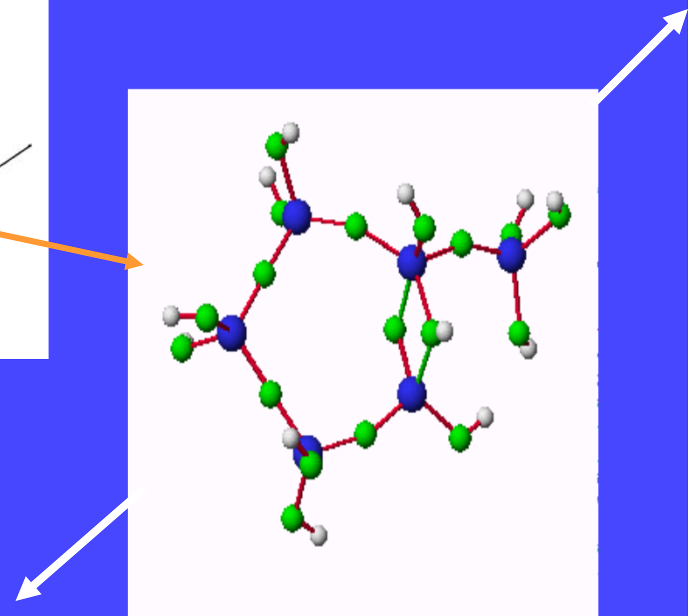


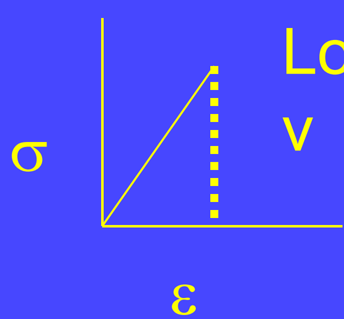
J. West et al., J. Non-Crystalline Solids 260 (1999) 99-108.

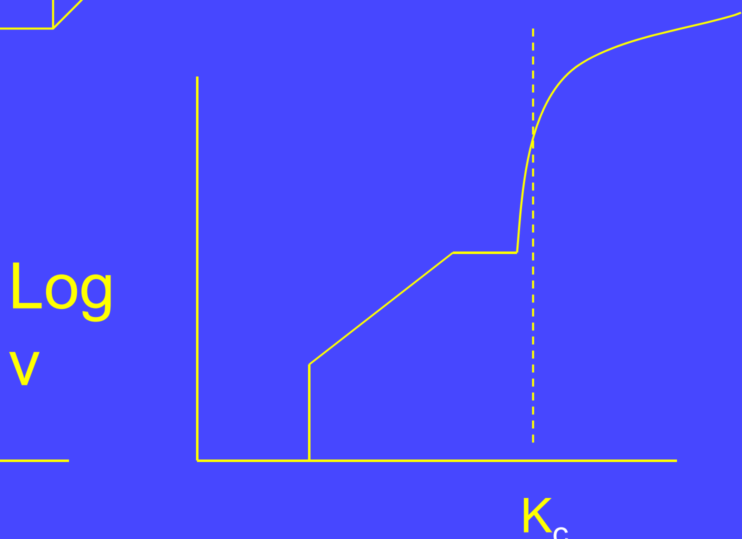
Bond Breaking Leads to Characteristic Features

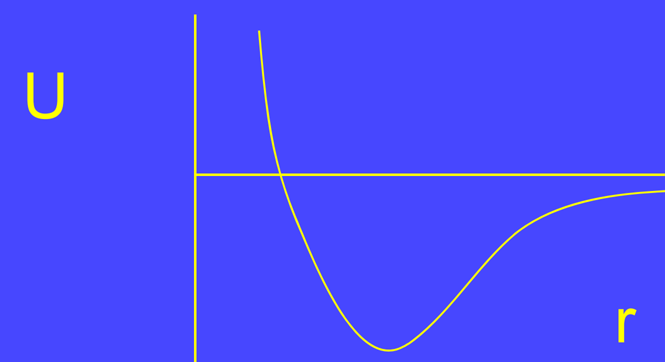






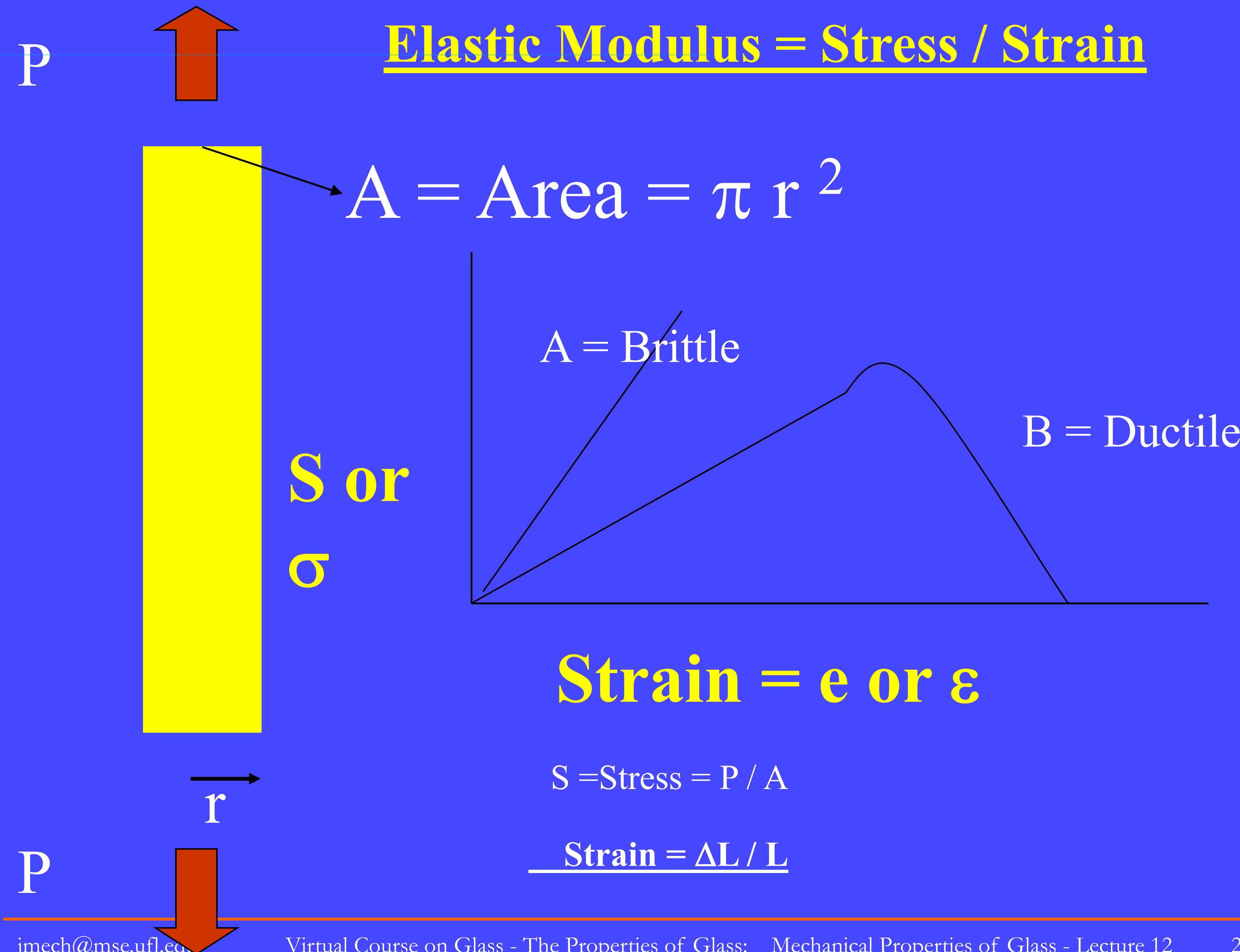






$$\text{Log } K = \text{Log } (Y\sigma\, c^{1/2})$$

Elastic Modulus = Stress / Strain



Observations show that strengths vary depending on ?

Strength of	common glass products	about 14–70 MPa
	freshly drawn glass rods	about 70–140 MPa
	abraded glass rods	about 14–35 MPa
	wet, scored glass rod	about 3–7 MPa
	armored glass	about 350–500 MPa
	handled glass fibers	about 350–700 MPa
	freshly drawn glass fibers	about 0.7–2.1 GPa

You have a few minutes to contemplate. Any ideas?

C. E. Inglis (1913) Suggested that flaws acted as stress concentrations

$\sigma_{yy} = \sigma_a[1+ (2c/b)]$

Radius at tip = ρ

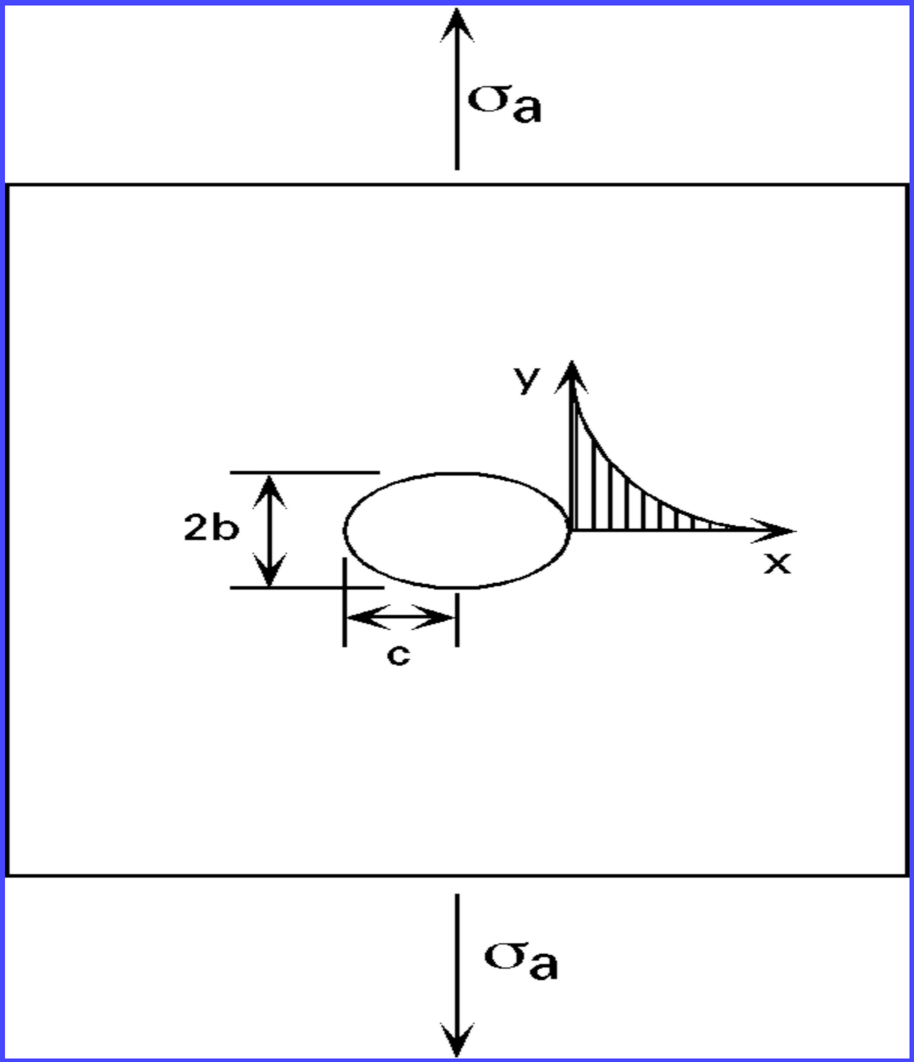
$\rho = b^2 / c$

$\sigma_{yy} = \sigma_a[1+ 2(c/\rho)^{1/2}]$

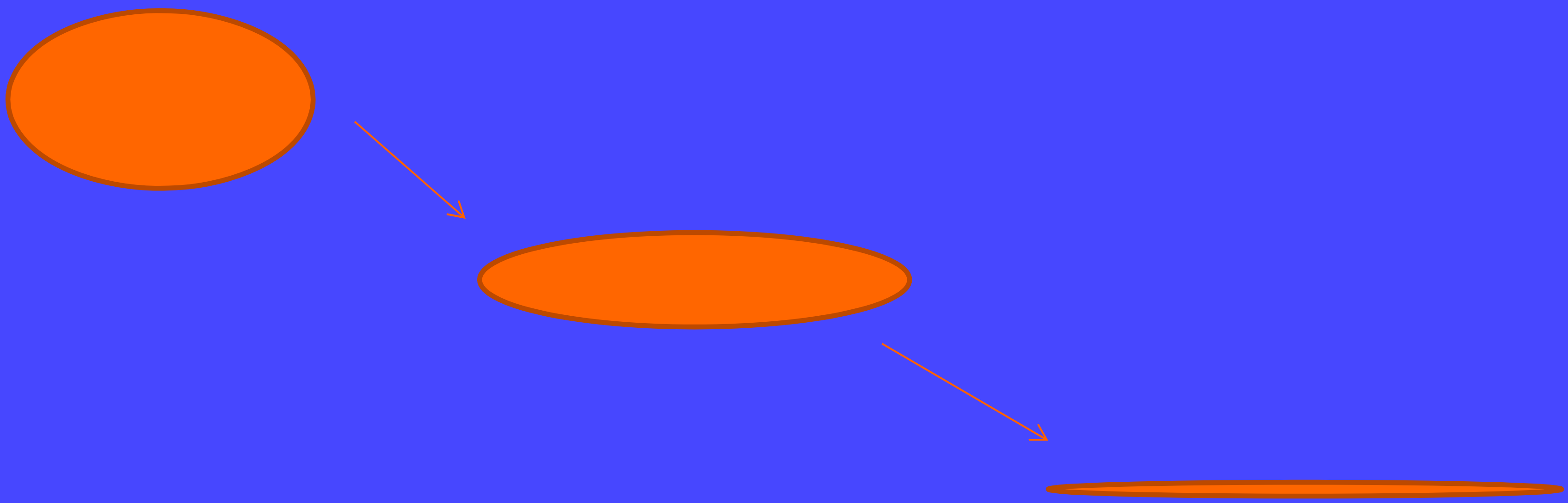
For flaws approaching the size of slit cracks, c >> ρ, and

$\sigma_{yy} \approx 2\ \sigma_a\ (c/\rho)^{1/2}]$

Also, $\rho \approx a_0$



An elliptical flaw in the limit can be thought of as a crack



If we follow that reasoning, then an elongated ellipse acts as a crack and we can calculate the strength -

$\rho \approx a_0$

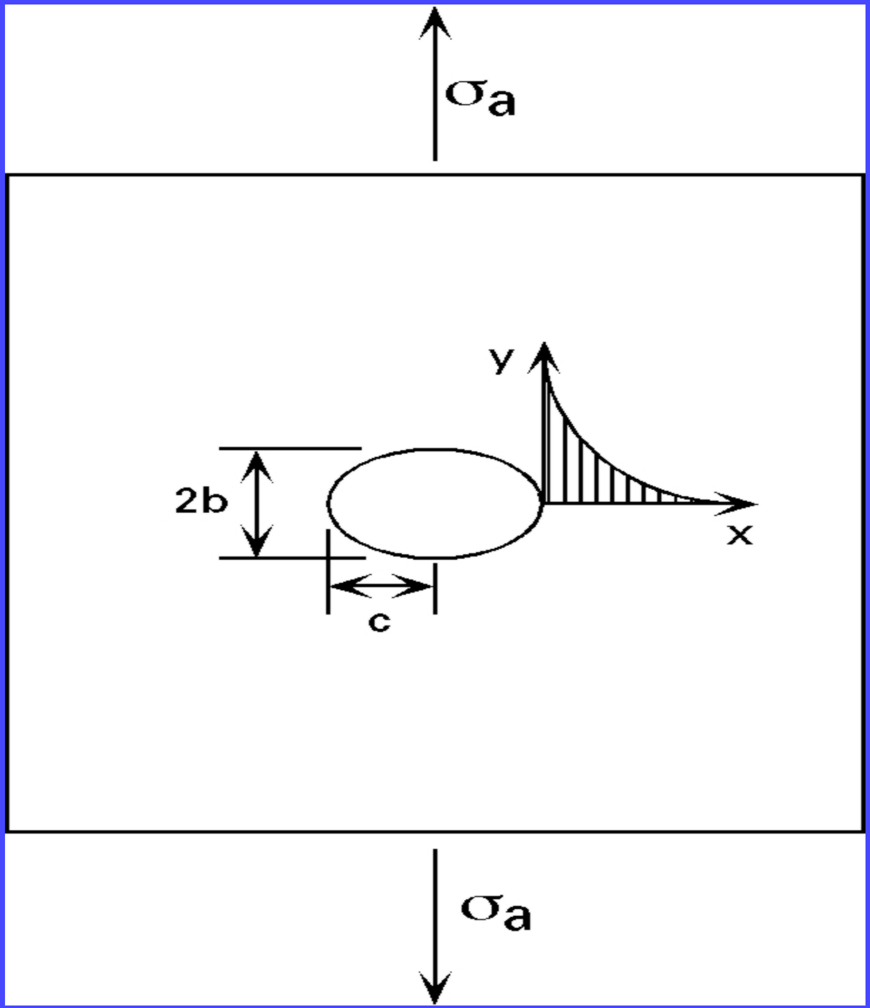
Theoretical strength:

$\sigma_m = [\gamma_f E / a_0]^{1/2}$

$\sigma_{yy} = \sigma_m$

$\sigma_f = [\sigma_a]_{\text{at failure}}$

$\sigma_f = (1/2)\, [\gamma_f E / c]^{1/2}$



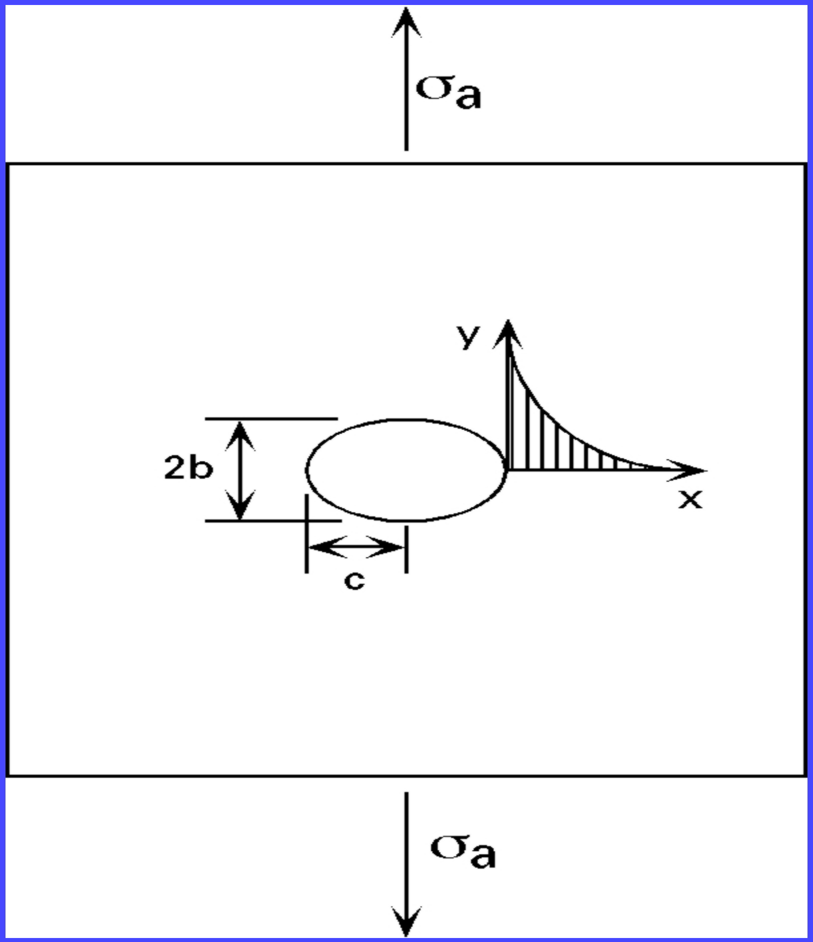
The calculated strength is much less than the 35 GPa calculated for the theoretical strength -

$$\sigma_f = (1/2)\,[\gamma_f E / c]^{1/2}$$

If E = 70 GPa, and γ_f = 3.5 J/m^2

Then a crack of 100 microns will result in a failure stress of

≈ 25 MPa

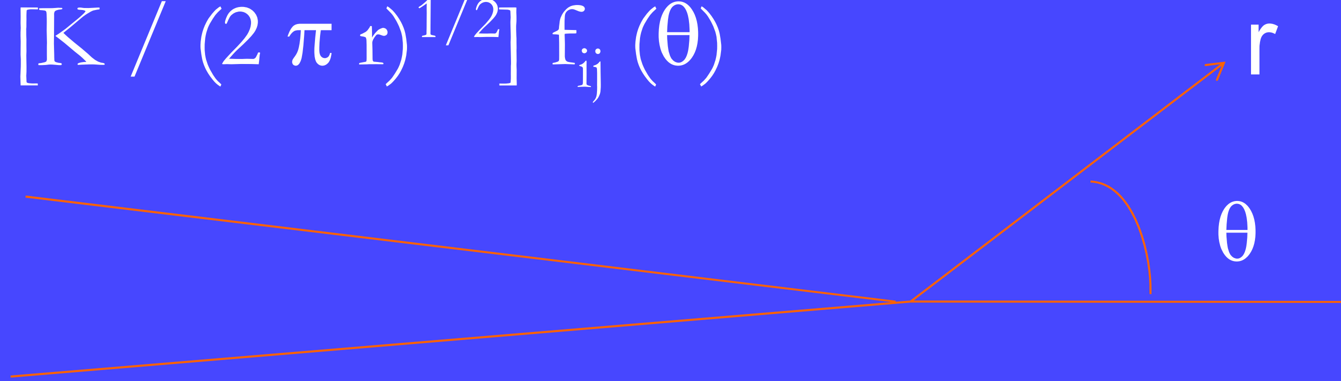


Following on Inglis' work, in 1921, A. A. Griffith was the first to suggest that low strengths observed were due to crack of a critical length, c*. He used an energy balance approach for a plate loaded in tension with a slit crack and arrived at what is now known as the Griffith equation:

$$\sigma_f = [2 \gamma_f E / \pi c^*]^{1/2}$$

In the 1950"s George Irwin introduced the concept of stress intensity based on an elasticity solution of Westergaard for a plate with a crack:

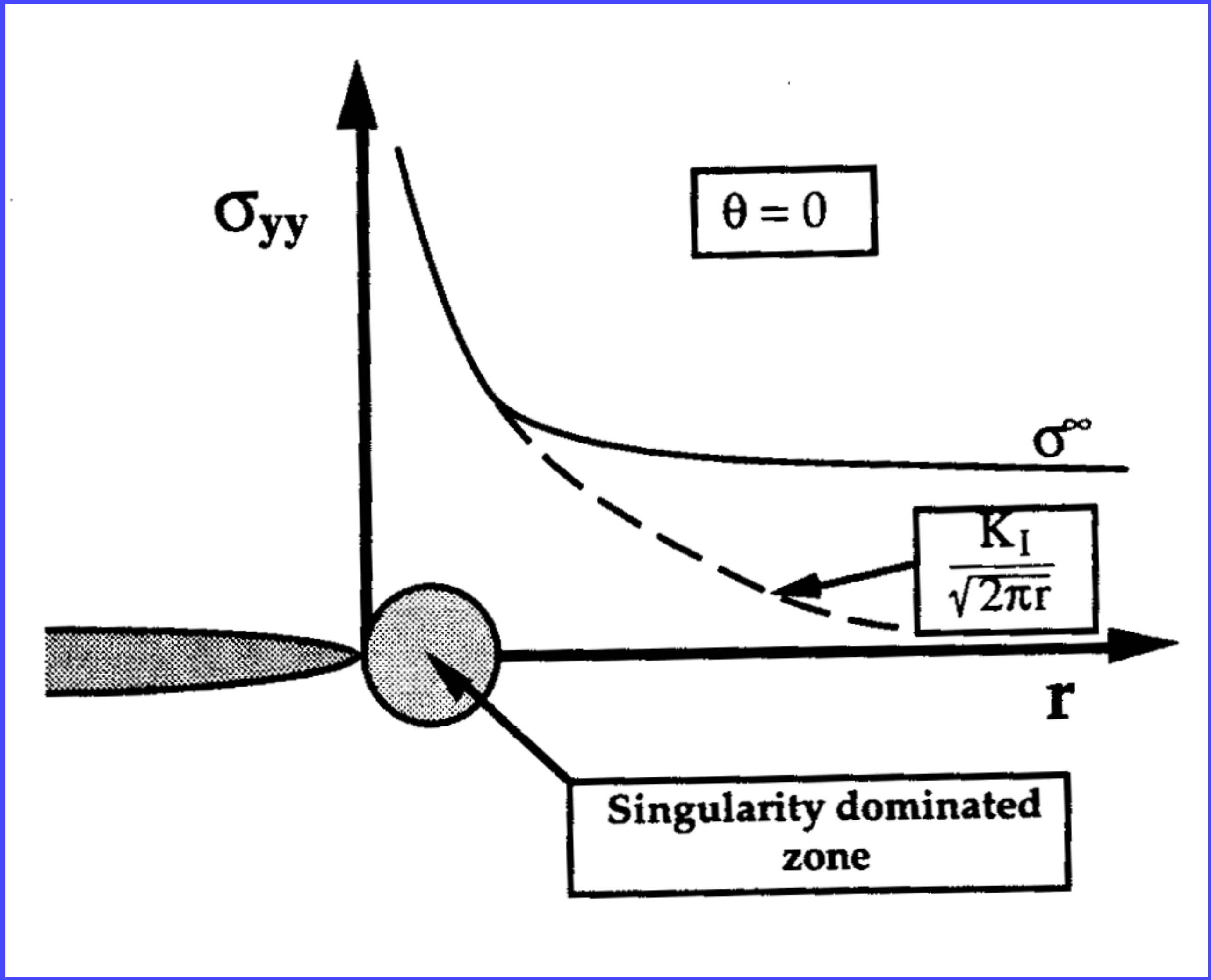
$$\sigma_{ij} = [K / (2 \pi r)^{1/2}] f_{ij} (\theta)$$



Irwin made the assumption that K is related to the far-field stress, σ_a :

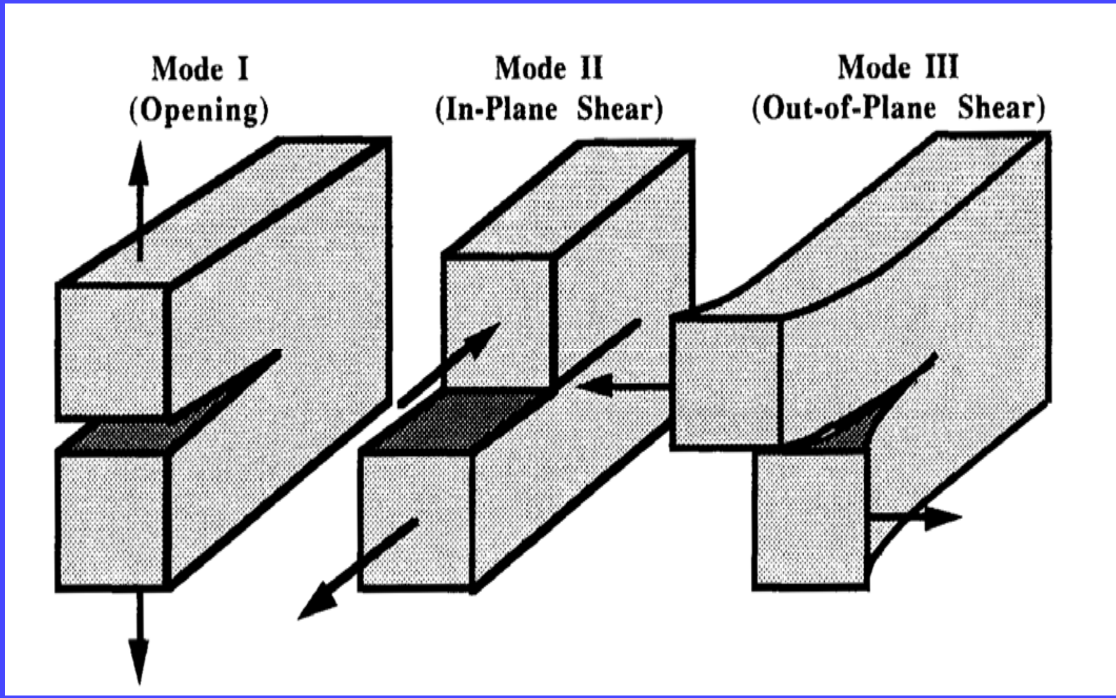
$$K = Y \sigma_a (c^{1/2})$$

As part of the solution, notice there is a stress singularity at r = 0.



There are three main Modes of loading:

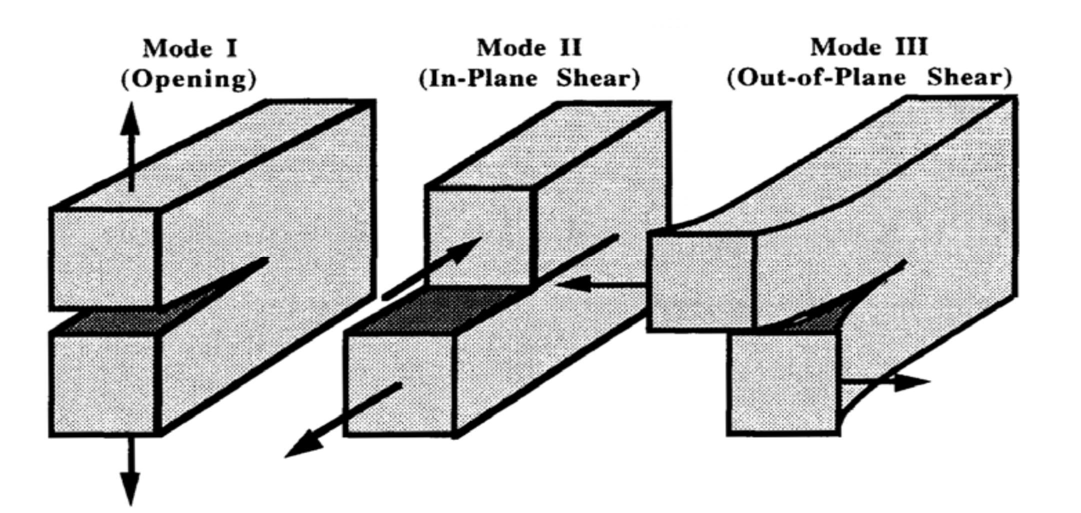
Mode I – tensile mode
Mode II – in-plane shear mode (sliding)
Mode III – out-of-plane shear (tearing mode)



$\sigma_{ij} = [K / (2 \pi r)^{1/2}] f_{ij} (\theta)$

Fracture in materials that fail in a brittle manner is governed by Mode I, i.e., fracture occurs in a plane perpendicular to the maximum principal tensile stress.

Note that loading can occur in a mixed mode manner.



The fracture criterion is based on stress intensity and on energy.

Irwin showed they are equivalent.

$$K_I \geq K_{IC} = Y \sigma c^{1/2}$$

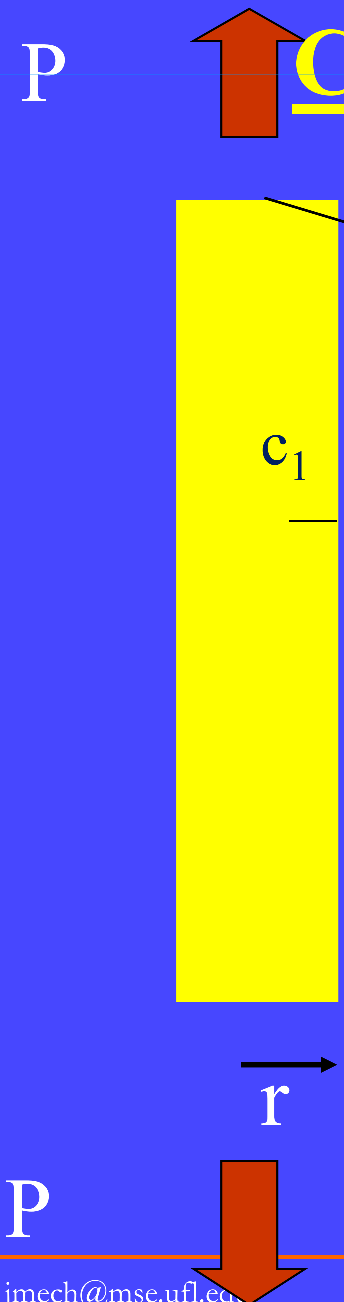
N.B. K_{IC} is pronounced K-one-cee

$$K_{IC} = [E' \mathcal{G}]^{1/2} = [E' (2\gamma_f)]^{1/2}$$

$\mathcal{G}$ is the strain energy release rate or crack extension force

$E' = E$	Plane Stress
$E' = E / (1-\nu^2)$	Plane Strain

Crack Size Governs Strength



$A = \text{Area} = \pi r^2$

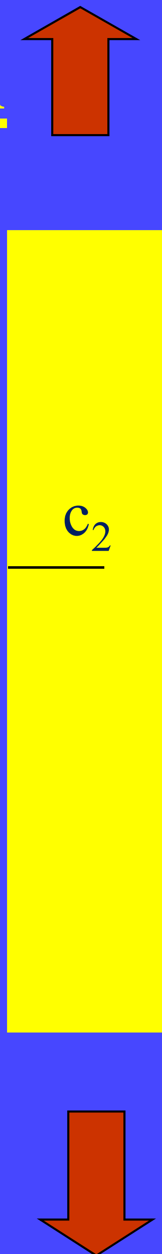
$K_c = \text{Toughness} = Y \sigma_1 c_1^{1/2}$

$K_c = \text{Toughness} = Y \sigma_2 c_2^{1/2}$

Strength = Stress at fracture

If $c_1 < c_2$ then $\sigma_1 > \sigma_2$

NOTE: Toughness Is Equal !



Toughness of a solid is a measure of its ability to adsorb energy prior to failure.

ASTM defines toughness nomenclature:

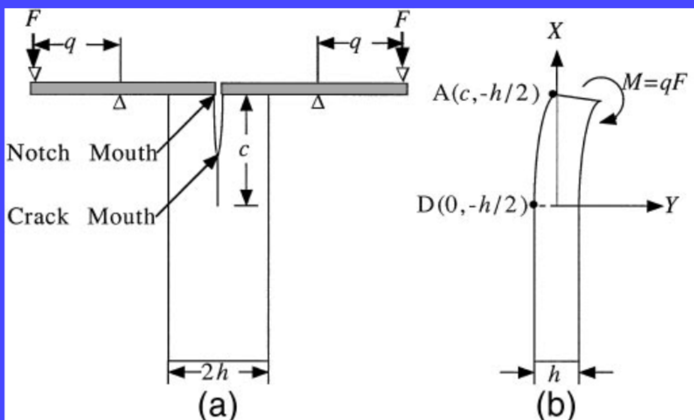
K_{IC} = fracture toughness

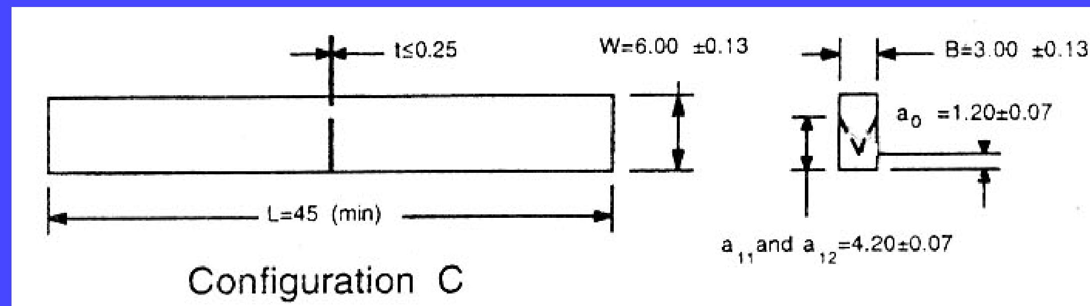
$\mathcal{G}_{IC}$ = toughness

γ_C = toughness

Different testing procedures can be used to obtain toughness

Large Crack Techniques





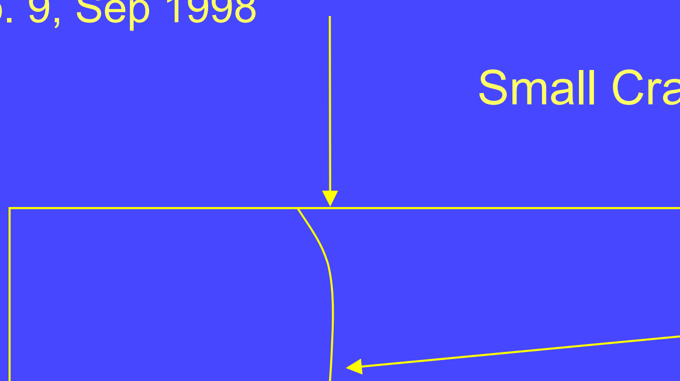
$\gamma = F^2 / f(\text{geom})$

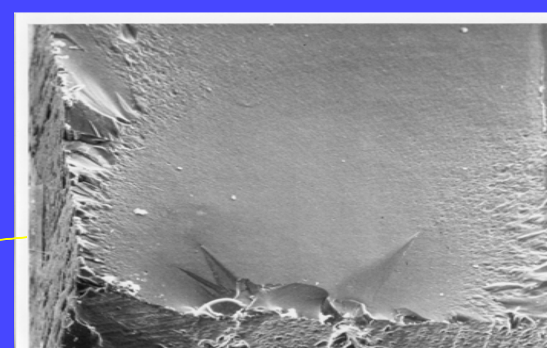
$K_c = Y \sigma (c)^{0.5}$

$K_c = [2 E' \gamma]^{½}$

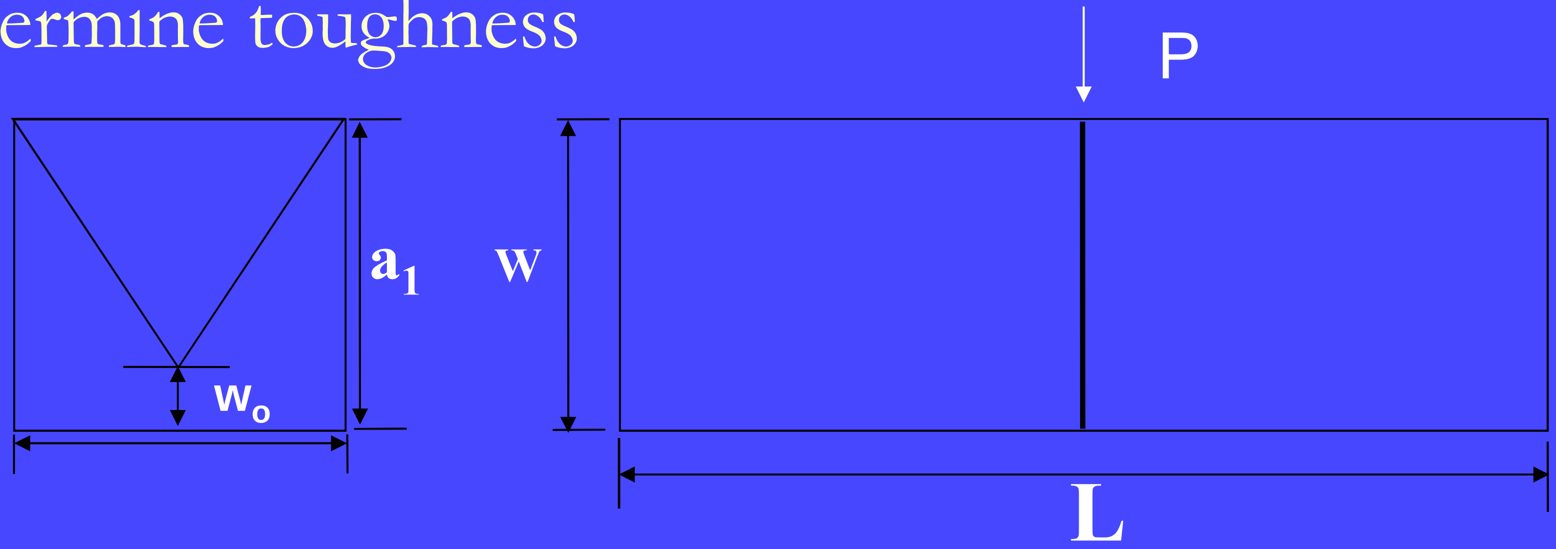
Hsueh et al. J. Mater. Res., Vol. 13, No. 9, Sep 1998

Small Crack Techniques





The Chevron Notch Specimen can be used to determine toughness



$$K_C = Y^*_{min}\left[\frac{P_{max}[S_o - S_i]10^{-6}}{BW^{3/2}}\right]$$

S_i = 0 for 3 point flexure

ASTM Standard C 1421-99, "Standard Test Method for Determination of Fracture Toughness of Advanced Ceramics at Ambient Temperature," ASTM International, West Conshohocken, PA.

Mechanical Strength Characterized By Loading In Biaxial Flexure

Strength for Monoliths

Monolithic Failure Stress Calculated From Failure Load :

$$\sigma_f = \frac{3P(1+\upsilon)}{4\pi\ t^2}\left[1 + 2\ln\frac{a}{b} + \left(\frac{1-\upsilon}{1+\upsilon}\right)\left(1 - \frac{b^2}{2a^2}\right)\frac{a^2}{R^2}\right]$$

where:

P = load at failure

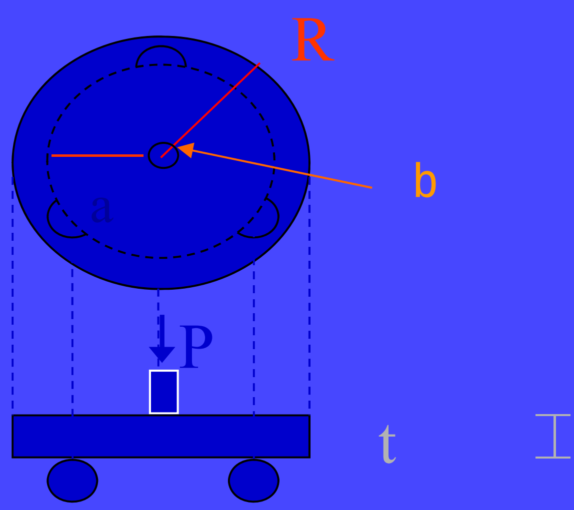
t = specimen thickness

a = support ring radius

b = loading piston radius

R= specimen radius

υ = Poison's ratio

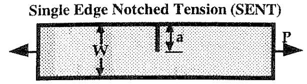
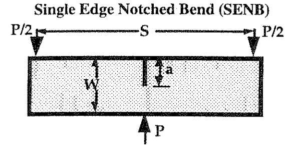
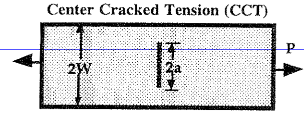
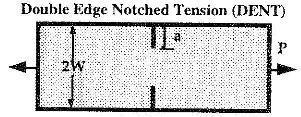
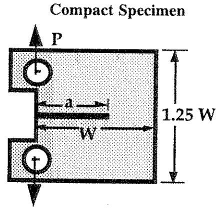


$K_{IC} = Y\,\sigma\,c^{1/2}$

Wachtman J.B., et.al. J. of Mater., 7 (2) 1972

There are many stress intensity solutions available

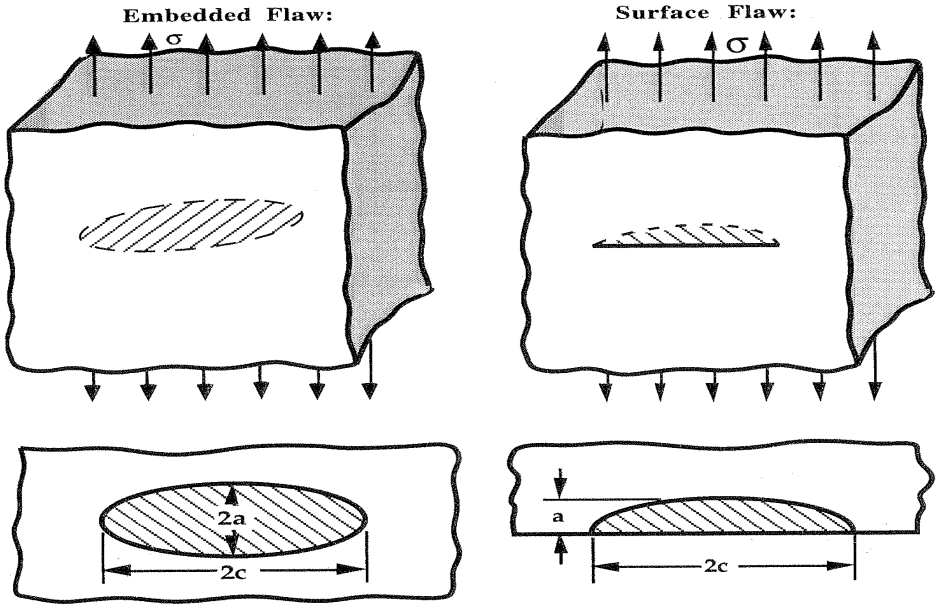
TABLE 2.4
K_I solutions for common test specimens [12].

GEOMETRY	$f(a/W)$*
Single Edge Notched Tension (SENT) 	$\dfrac{\sqrt{2\tan\frac{\pi a}{2W}}}{\cos\frac{\pi a}{2W}}\left[0.752+2.02\left(\frac{a}{W}\right)+0.37\left(1-\sin\frac{\pi a}{2W}\right)^3\right]$
Single Edge Notched Bend (SENB) 	$\dfrac{3\frac{S}{W}\sqrt{\frac{a}{W}}}{2\left(1+2\frac{a}{W}\right)\left(1-\frac{a}{W}\right)^{3/2}}\left[1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left\{2.15-3.93\left(\frac{a}{W}\right)+2.7\left(\frac{a}{W}\right)^2\right\}\right]$
Center Cracked Tension (CCT) 	$\sqrt{\frac{\pi a}{4W}\sec\frac{\pi a}{2W}}\left[1-0.025\left(\frac{a}{W}\right)^2+0.06\left(\frac{a}{W}\right)^4\right]$
Double Edge Notched Tension (DENT) 	$\dfrac{\sqrt{\frac{\pi a}{2W}}}{\sqrt{1-\frac{a}{W}}}\left[1.122-0.561\left(\frac{a}{W}\right)-0.205\left(\frac{a}{W}\right)^2+0.471\left(\frac{a}{W}\right)^3+0.190\left(\frac{a}{W}\right)^4\right]$
Compact Specimen 	$\dfrac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{3/2}}\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.60\left(\frac{a}{W}\right)^4\right]$

$* K_I = \frac{P}{B\sqrt{W}} f(a/W)$ where B is the specimen thickness.

e.g., Crack Tip Stress Fields, R. J. Sanford, ed. SEM Classic Papers V. CP 2. (1997)

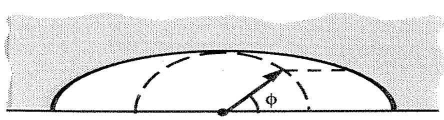
Several crack shapes are common:



$$K_I = \sigma\sqrt{\frac{\pi a}{Q}}\, f(\phi)$$

$$K_I = \lambda_s \sigma\sqrt{\frac{\pi a}{Q}}\, f(\phi)$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$



$$\lambda_s = \left[1.13 - 0.09\left(\frac{a}{c}\right)\right]\left[1 + (1 - \sin\phi)\right]^2$$

$$f(\phi) = \left[\sin^2\phi + \left(\frac{a}{c}\right)^2 \cos^2\phi\right]^{\frac{1}{4}}$$

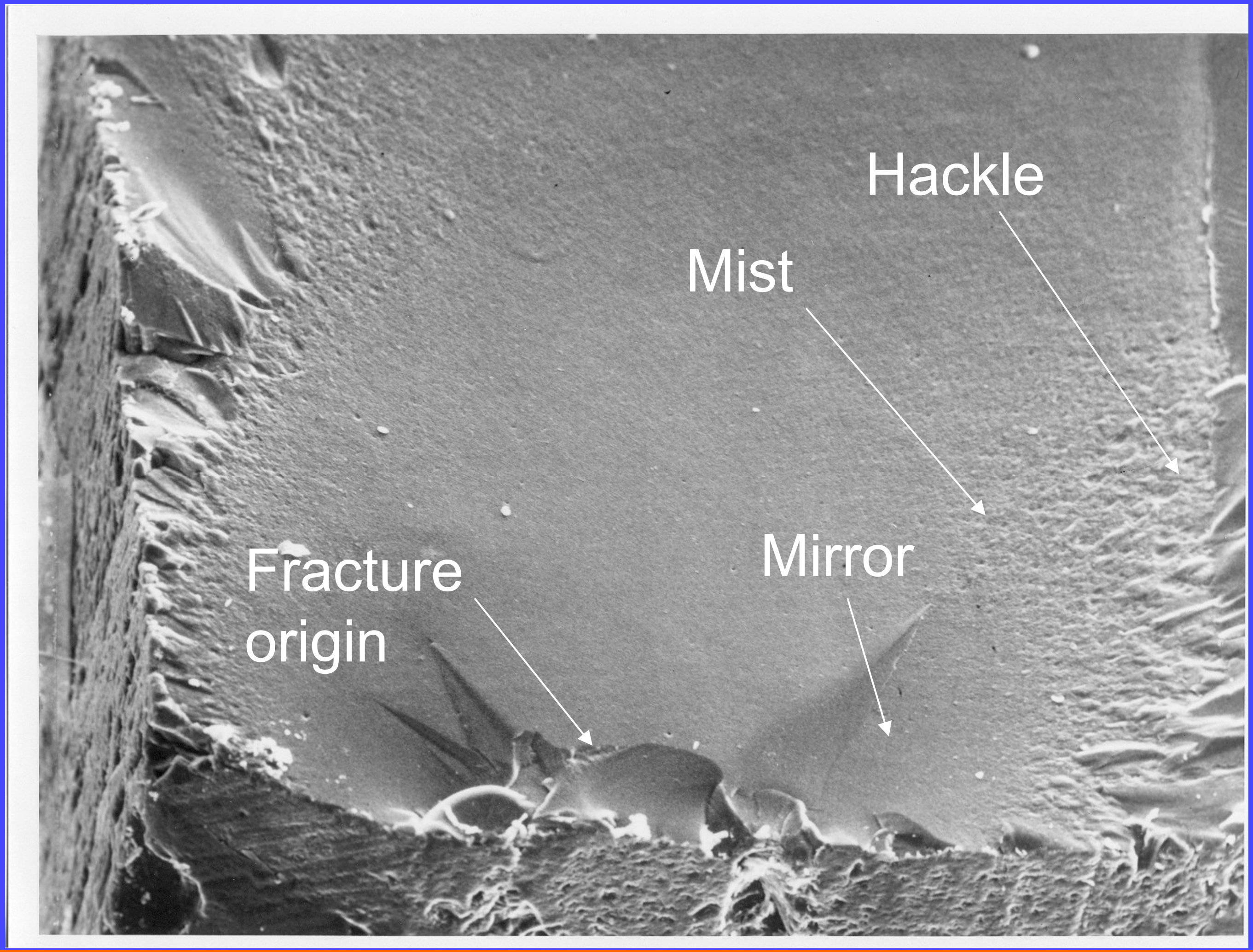
FIGURE 2.19 Mode I stress intensity factors for elliptical and semi-elliptical cracks. These solutions are valid only as long as the crack is small compared to the plate dimensions and $a \le c$.

Mechanical Properties of Glass

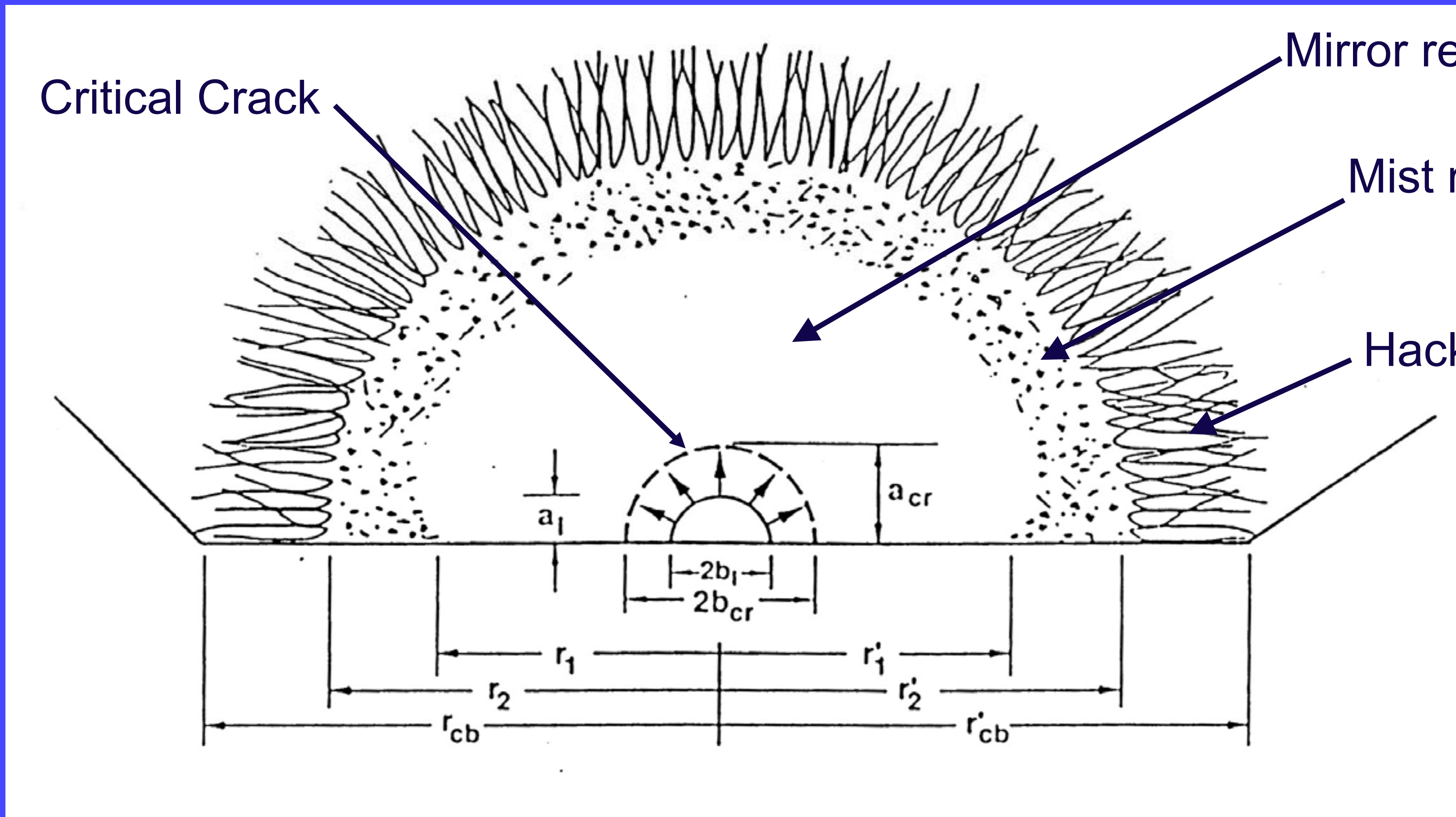
- Elastic Modulus and Microhardness
  [Chapter 8 – The "Good Book"*]
- Strength and Toughness [Chapter 18]
  - Fracture mechanics tests
  - **Fractography**
  - Stress Corrosion
  - Fracture Statistics

*A. Varshneya, "Fundamentals of Inorganic Glasses", Society of Glass Technology (2006)

Characteristic Markings Are Observed on the Fracture Surface



Characteristic Features Aid Failure Analysis



$K_C = Y\,\sigma\,(c)^{1/2}$

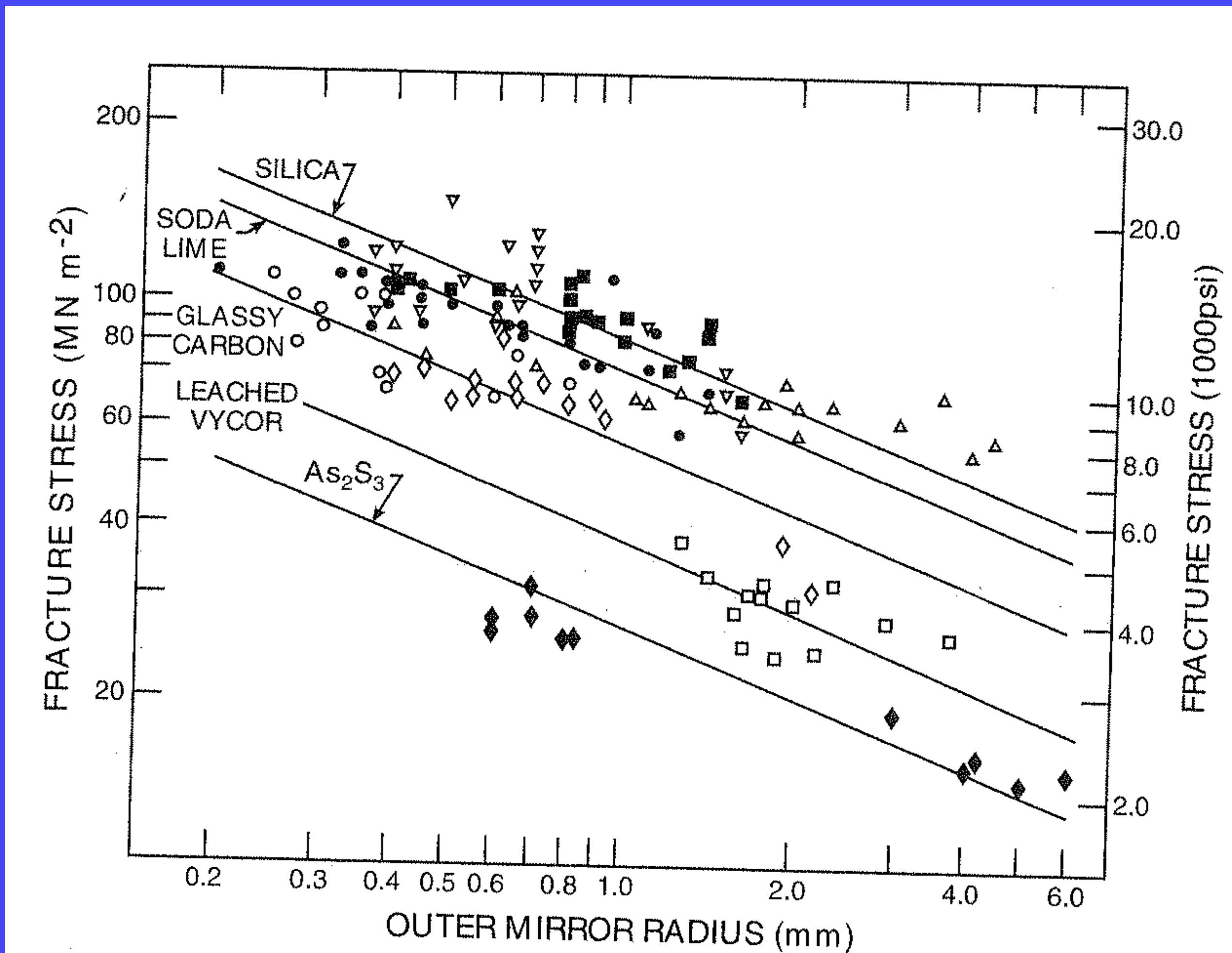
$c = (a\,b\,)^{1/2}$

$K_{Bj} = Y\,\sigma\,(r_j)^{1/2}$

$r_j\,/\,c = \text{constant}$

Mirror constants are related to toughness of materials

$\log \sigma_f = \log M_2 - 0.5 \log r_2$



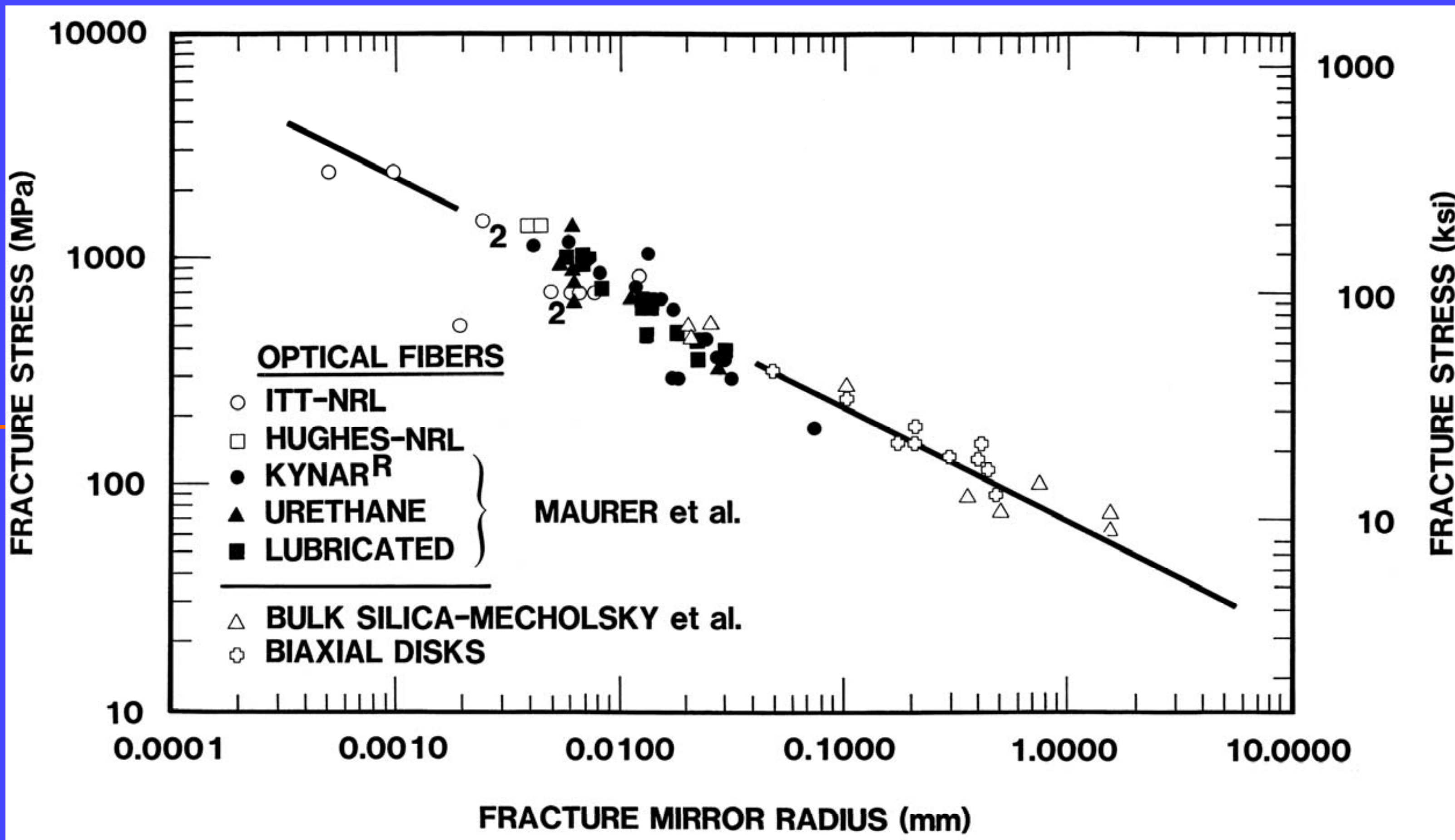
$\sigma_f = M_2 / r_2^{0.5}$

$\sigma_f = K_{B2} / Y_2\, r_2^{0.5}$

J. J. Mecholsky, R.W. Rice and S. W. Freiman, JACerS 57, 440 (1974)

Relationship Holds For Large Size & Stress Range

$\sigma r^{1/2} = \text{constant}$



J.J. Mecholsky, Jr., Fractography of Optical Fibers, in ASM Engineered Materials Handbook, 4, Ceramics and Glasses, Section 9: Failure Analysis, (1992).

Fracture Mechanics & Fractography Provide A Framework for Quantitative Analysis

$K_{IC} = Y \sigma c^{1/2}$ Crack Boundary

$K_{B1} = Y_1 \sigma r_1^{1/2}$ Mirror-Mist Boundary

$K_{B2} = Y_2 \sigma r_2^{1/2}$ Mist-Hackle Boundary

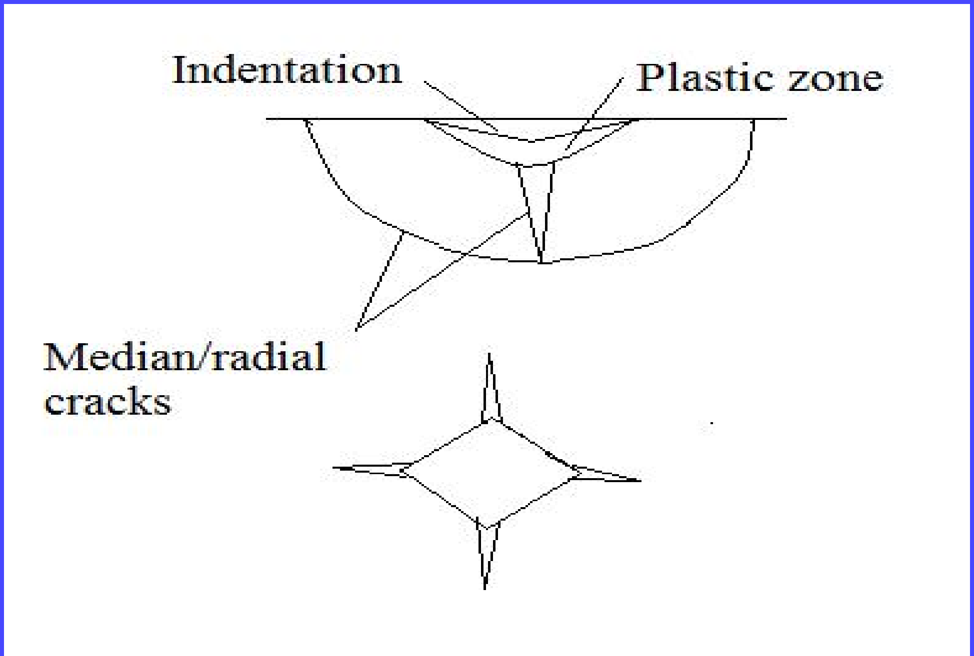
$K_{B3} = Y_3 \sigma r_3^{1/2}$ Crack Branching Boundary

[c/r_j = constant]

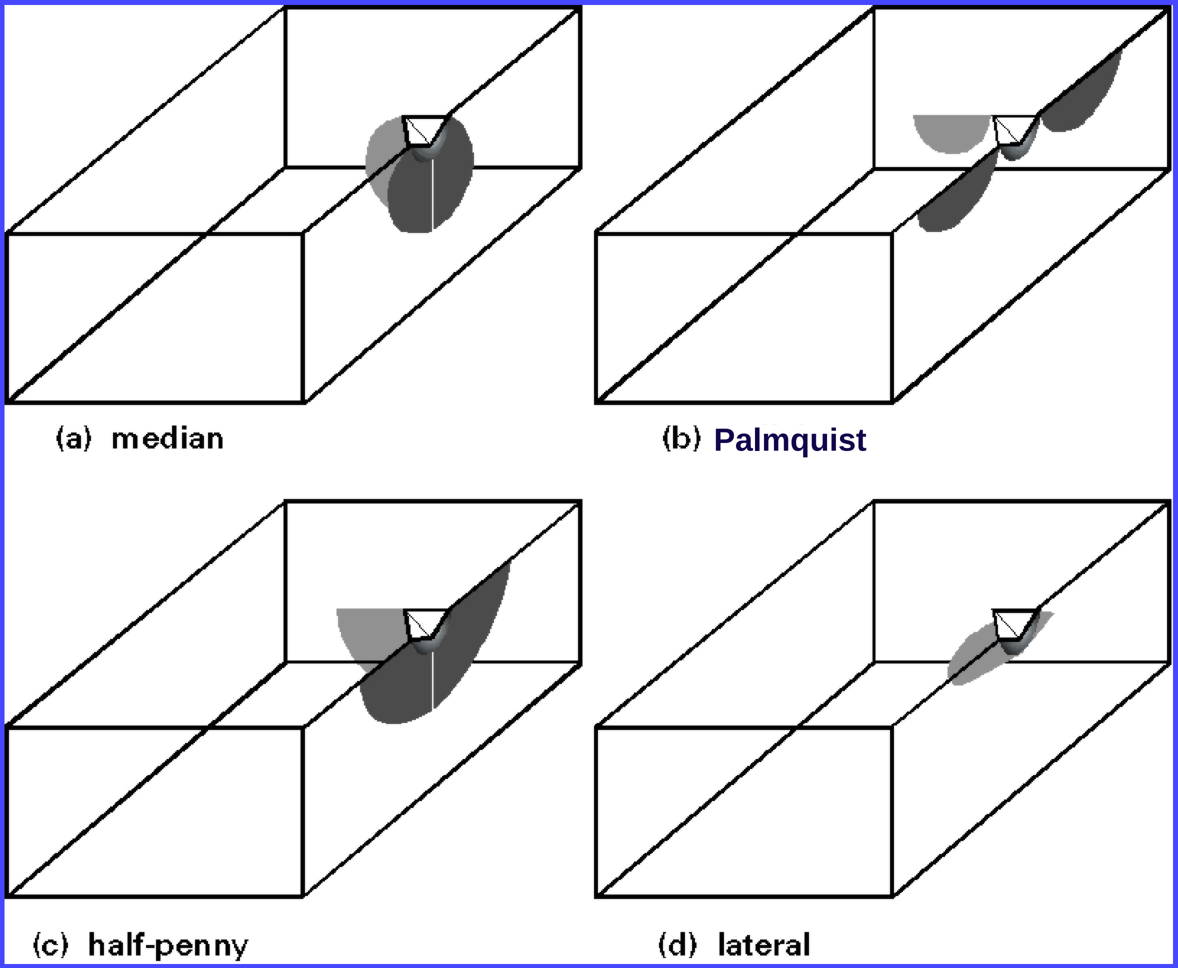
Hardness Indentation Can Be Used To Measure Toughness

N.B. : small crack technique

- A hardness indent is made on the sample with a diagonal length 2a and a system of radial cracks with total length 2c using a load P.



Overloaded indentation leads to a crack system



The hardness and elastic modulus are related to toughness using the indentation technique

Hardness is given by :

$$H = P/\alpha\, a^2$$

The value of α depends on the shape of the indenter and is equal to 2 for a Vickers indenter.

The critical stress intensity for crack propagation:

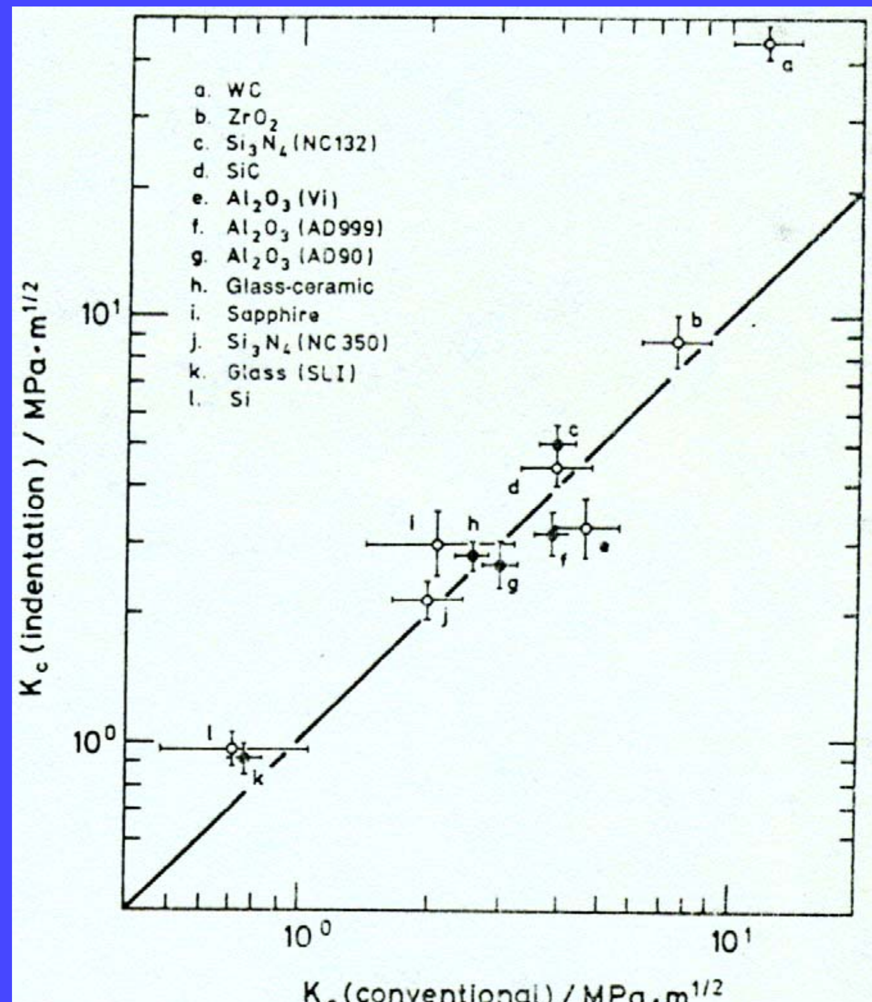
$$K_{IC} = \frac{\zeta\, (E/H)^{1/2}\, P}{c^{3/2}}$$

Studies on many ceramics led to an average value of

$$\zeta = 0.016 \pm 0.004$$

The crack indentation method generally agrees with "conventional" fracture toughness values

$$K_{IC} = \underline{\zeta\,(E/H)^{1/2}\,P}c^{-3/2}$$



Strength indentation method does not require crack size

- Measure strength after indentation.

- Precautions should be taken to prevent the crack from elongating by slow crack propagation during the time between indentation and strength measurement.

The critical stress intensity for crack propagation is given by:

$$K_{IC} = \eta \, (E/H)^{1/8} \, (\sigma_m \, P^{1/3})^{3/4}$$

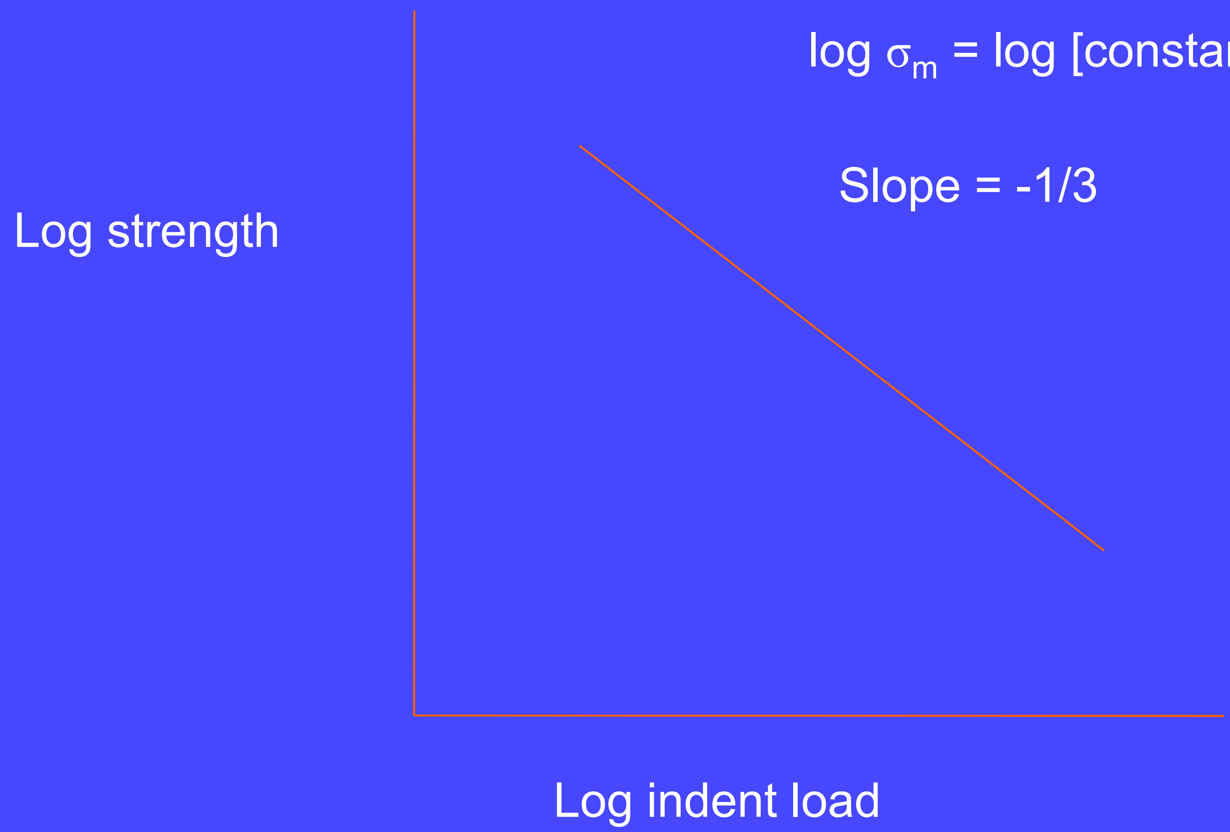
Studies on many ceramics have led to an average value of

$$\eta = 0.59 \pm 0.12$$

Strength indentation method provides toughness and does not require crack length

$$K_{IC} = \eta\,(E/H)^{1/8}\,(\sigma_m\,P^{1/3})^{3/4}$$

$$\log \sigma_m = \log[\text{constant}] - 1/3 \log P$$



Mechanical Properties of Glass

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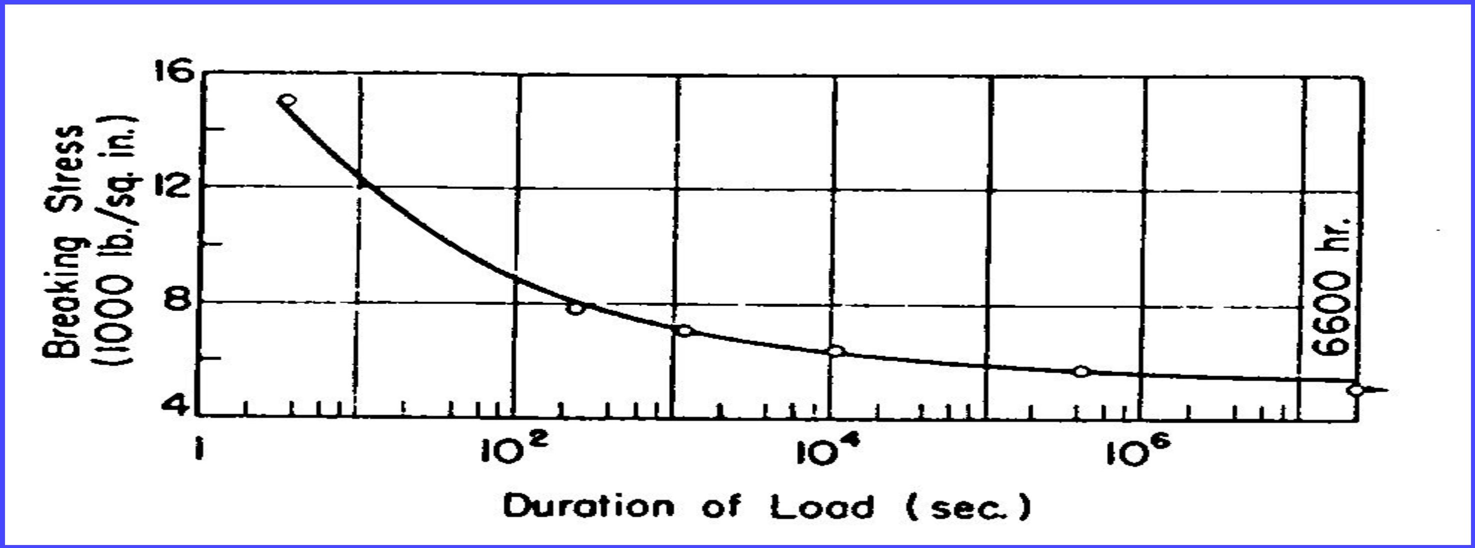
  - Fracture Statistics

*A. Varshneya, “Fundamentals of Inorganic Glasses”, Society of Glass Technology (2006)

Early investigators observed the time dependence of the strength of glass



Stress-time characteristics of glass, from bending tests on 1/4 inch diameter soda-lime-silicate rods

E.B. Shand, "Experimental Study of Fracture of Glass: I, The Fracture Process," *J. Am. Ceram. Soc.* **37**, 52 (1954); original figure from C.J. Phillips, "Mechanical Strength of Glass";report, Research Laboratory, Corning Glass Works, 1937.

In 1947 Gurney presented thermodynamic concepts to explain moisture enhanced crack growth

"Due to concentration of strain energy, the material at the end of the crack has a much higher free energy than normal unstressed glass, and is therefore much more chemically active. Atmospheric attack will result in the formation of a complex of glass and atmospheric constituents. The crack will extend continually if the strength of this complex, during or after its formation, is less than the load imposed on it."

C. Gurney and S. Pearson, "The Effect of the Surrounding Atmosphere on the Delayed Fracture of Glass," *Proc. Phys. Soc B*, **62** 469-476 (1949).

Abrasion decreases the strength of glass. Time under load decreases strength of glass.

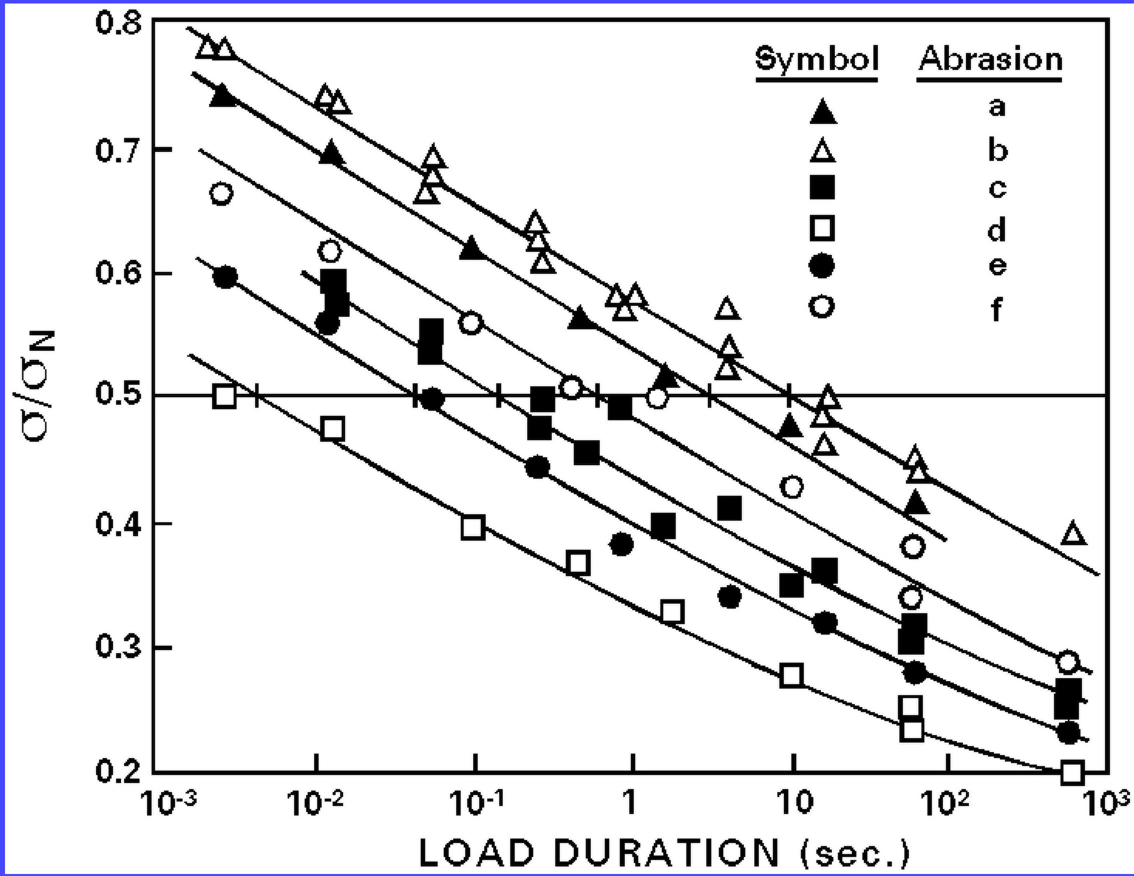


Fig. 18-4

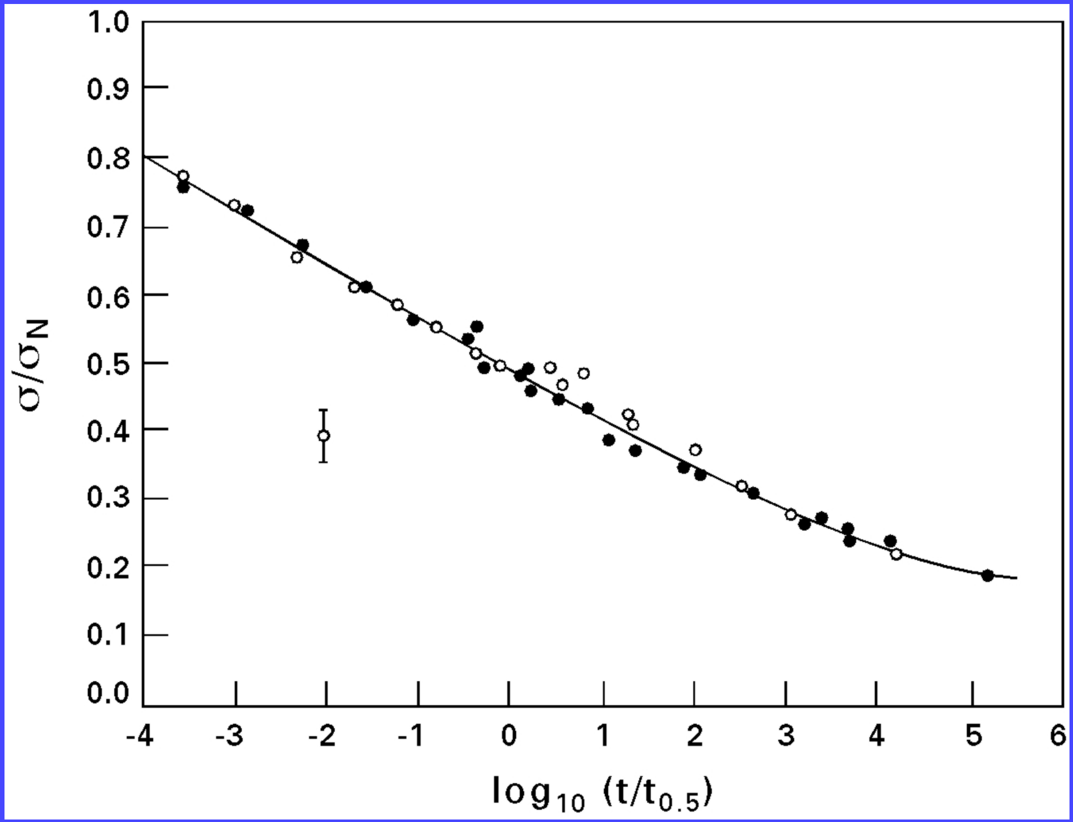
R.E. Mould and R. D. Southwick ,JACerS 42,542-547&582-592 (1959).

Mould and Southwick showed that cracks grow in time with applied stress

$$\sigma/\sigma_n = - A \log(t/t_{0.5}) + B$$

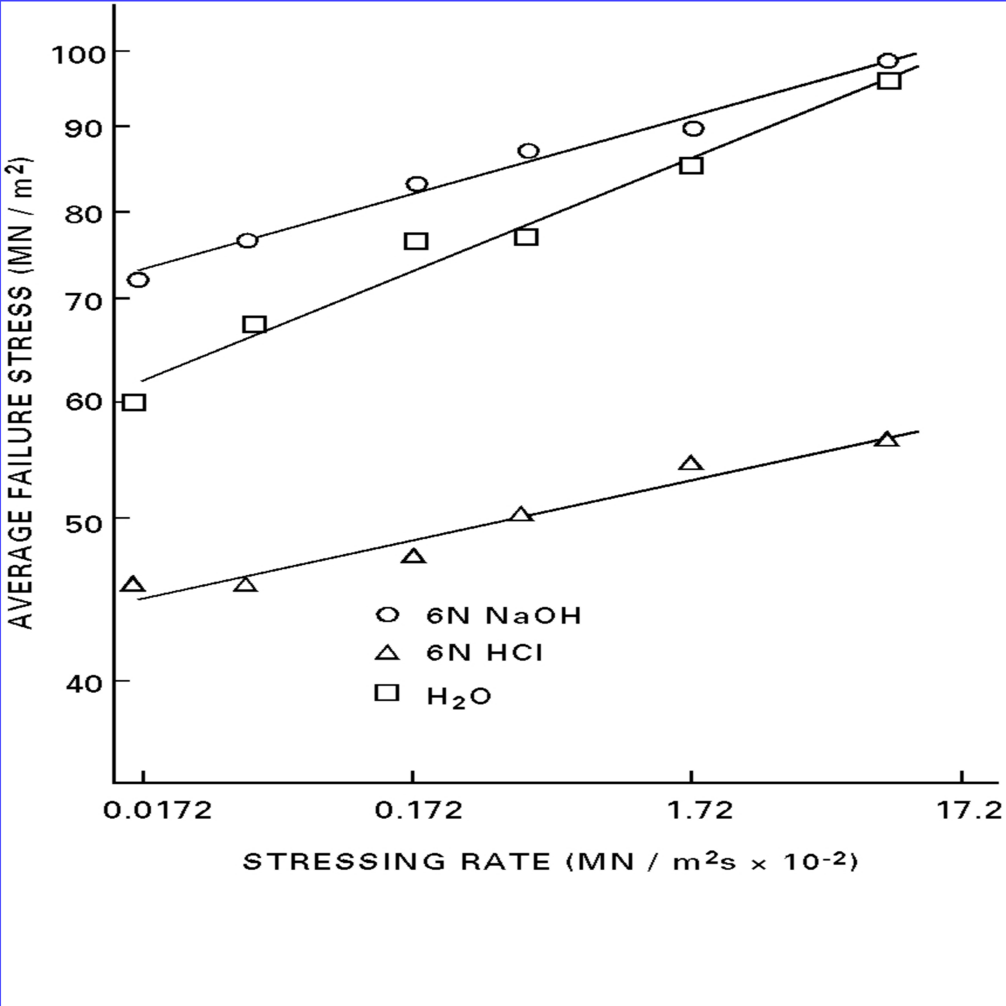
"Universal "static fatigue curve

Fig. 18-5



R.E. Mould, "The Strength of Inorganic Glasses," pp. 119 to 149 in *Fundamental Phenomena in the Materials Sciences, V. 4: Fracture of Metals Polymers and Glasses,* Edited by L.J. Bonis, J.J. Duga and J.J. Gilman Plenum Press, New York (1967).

Greater stressing rates increase strength

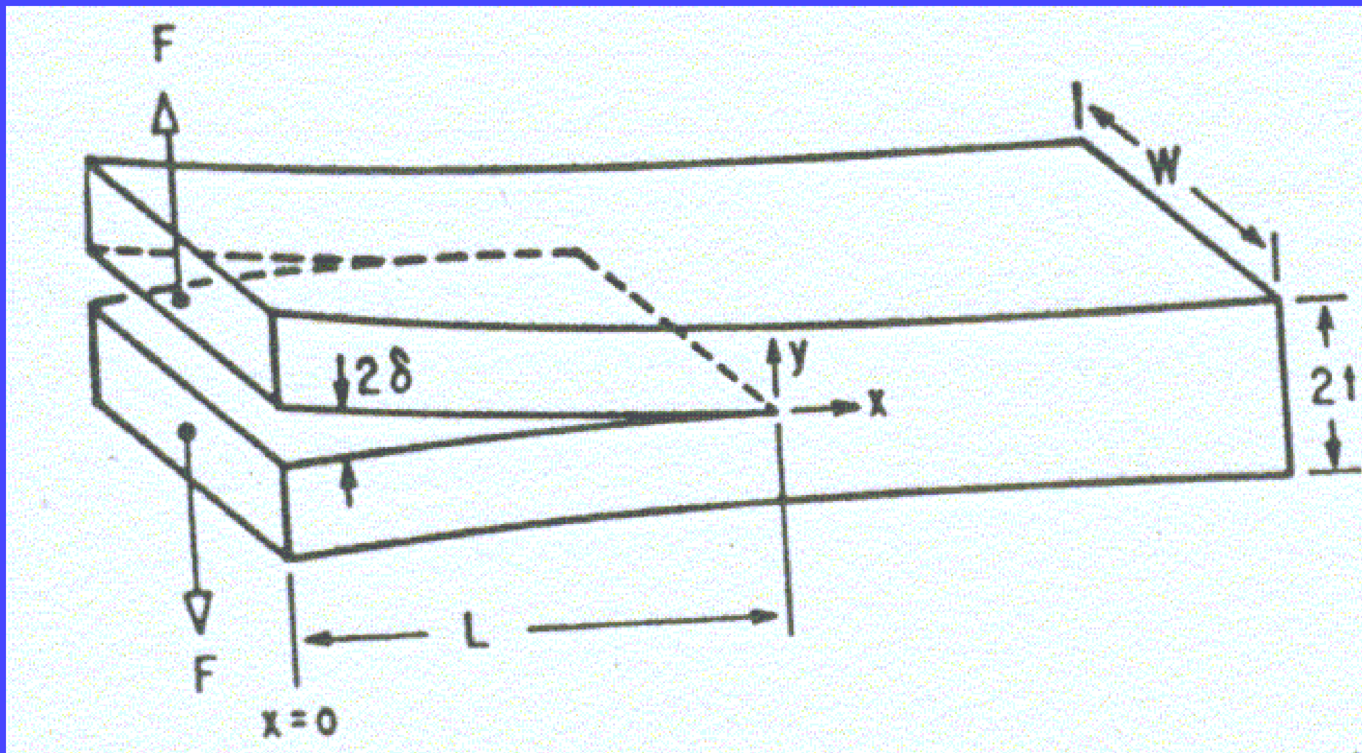


$$\sigma_f^{n+1} = 2(n+1)\left[\frac{d\sigma_a}{dt}\right]\left[\frac{\sigma_{Ic}^{n-2}}{(n-2)AY^2K_{Ic}^{n-2}}\right] = B\left[\frac{d\sigma_a}{dt}\right]$$

Fig. 18-6

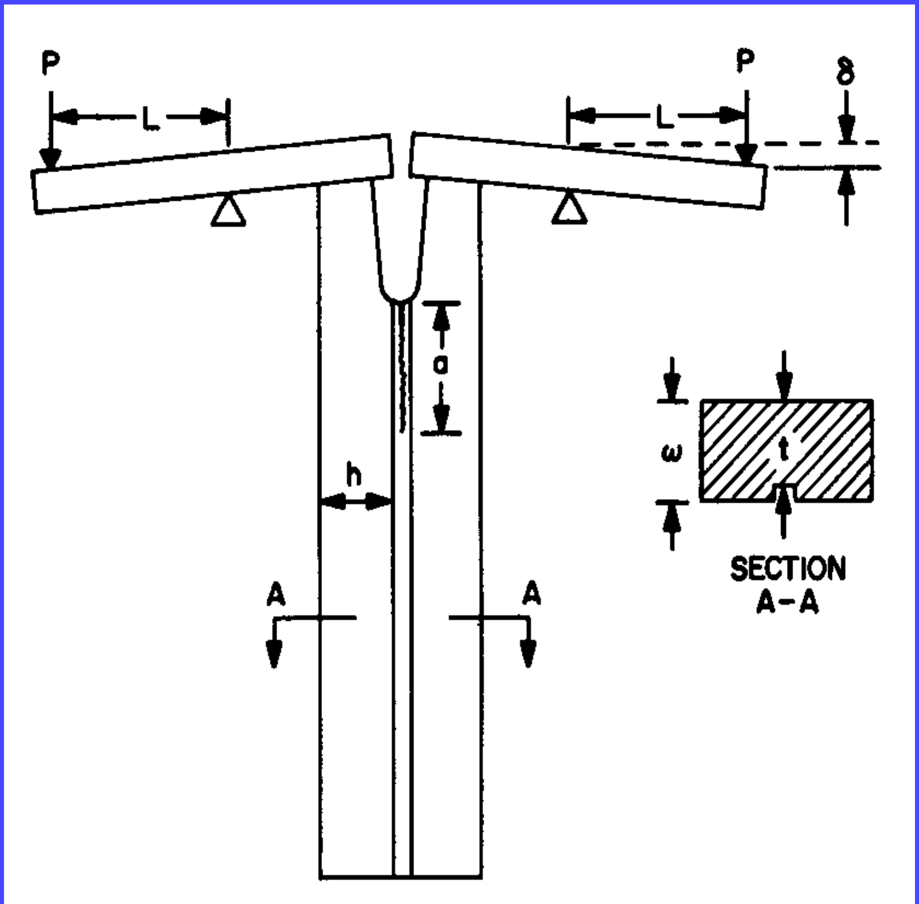
JACerS 58 (7-8) 265-67 (1975)

In order to understand crack growth with time, you need to measure crack growth directly.



$$\gamma_A = \frac{F^2 L^2}{2EIt}$$

The constant moment DCB is often used.



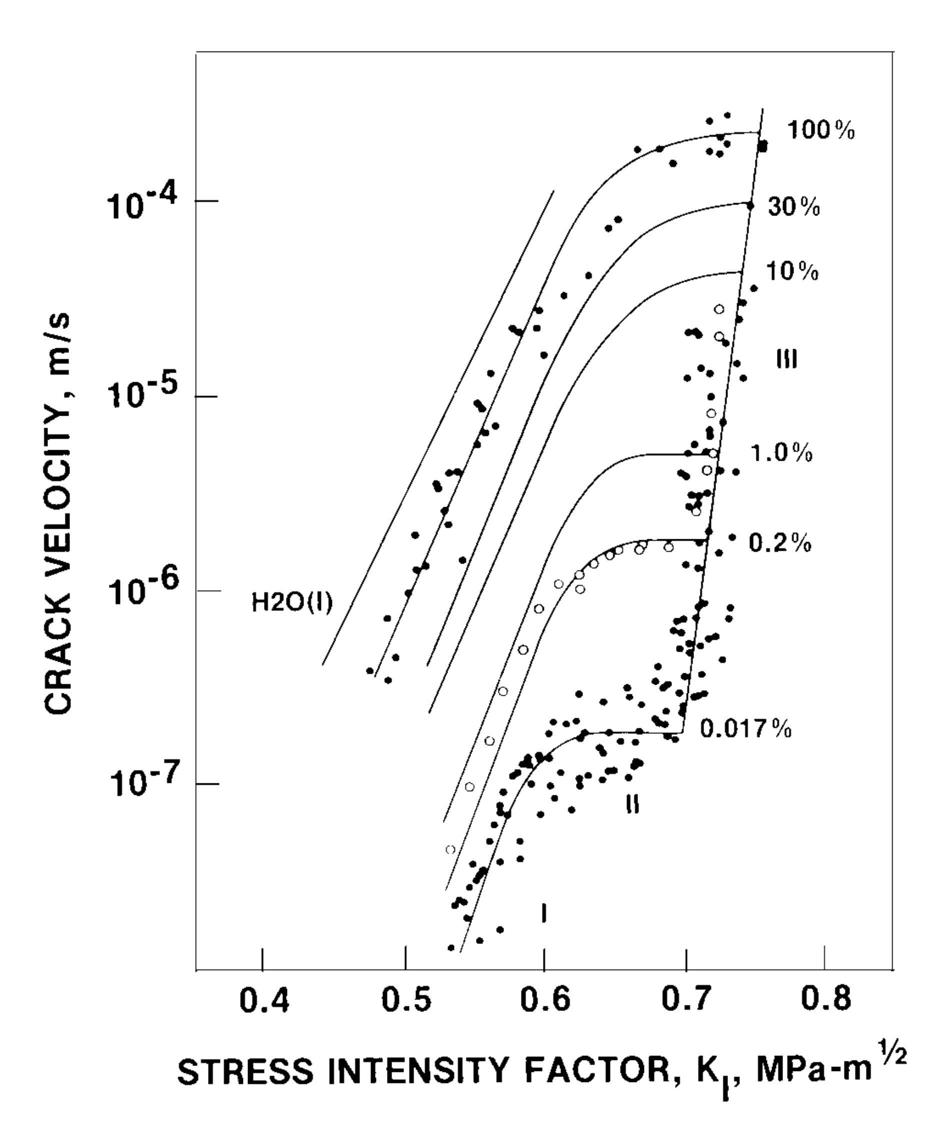
v = dc/dt

$K_I = P^2 / f(\text{geometry})$

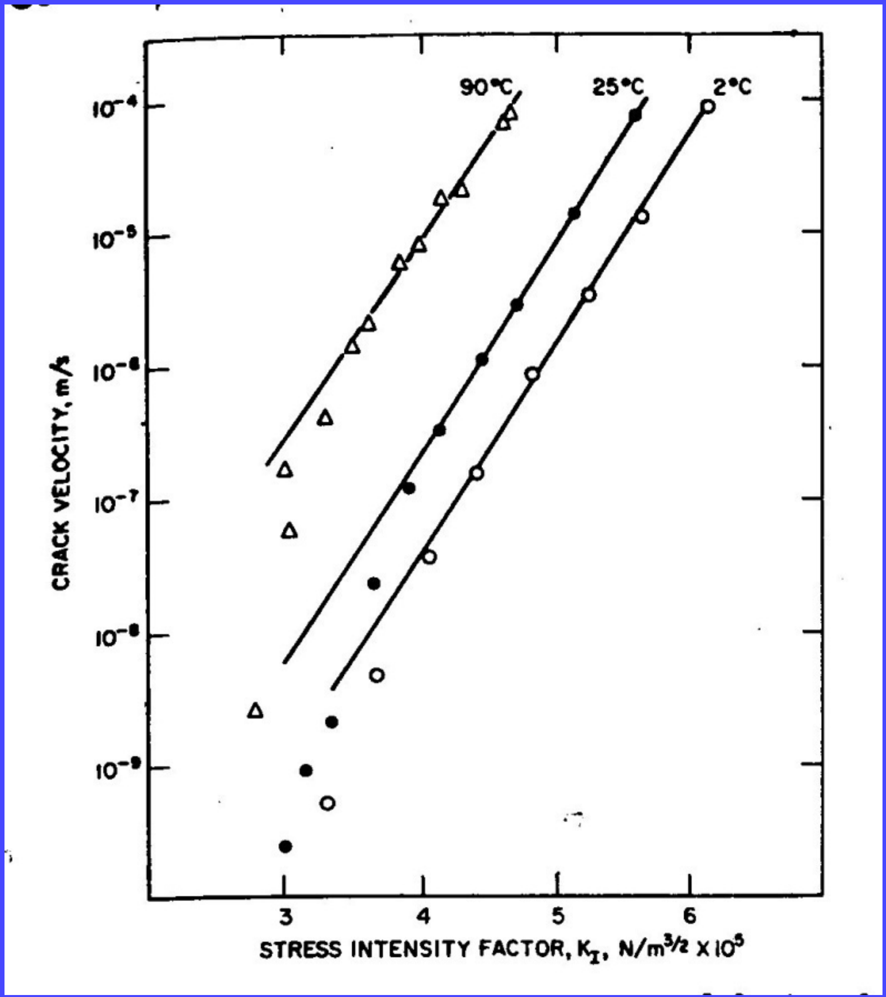
S. W. Freiman[1], D. R. Mulville[1] and P. W. Mast[1] J. Materials Science V. 8, Number 11 / November, 1973 1573-4803

Water and stress enhance crack growth in glass

S.M. Wiederhorn, "Influence of Water Vapor on Crack Propagation in Soda-Lime Glass,"
J. Am. Ceram. Soc. **50** [8] 407-14 (1967).



Some glasses show "static" fatigue limits



Glass tested in water.
Note the two different kinds of behavior – glasses containing alkali ions exhibit apparent fatigue limits; glasses with no alkali ions form straight lines on this kind of graph.

Slow crack growth is a thermally activated process

There is still uncertainty over the exact form of the $V - K_I$ curves. It is clear from chemical rate theory that an exponential expression is fundamental, namely:

$$V = V_0 \exp(-E + bK_I / RT)$$

where V_0 is a constant, E is the activation energy for the reaction, R is the gas constant, T is the temperature, and b is proportional to the activation volume for the crack growth process, ΔV^*.

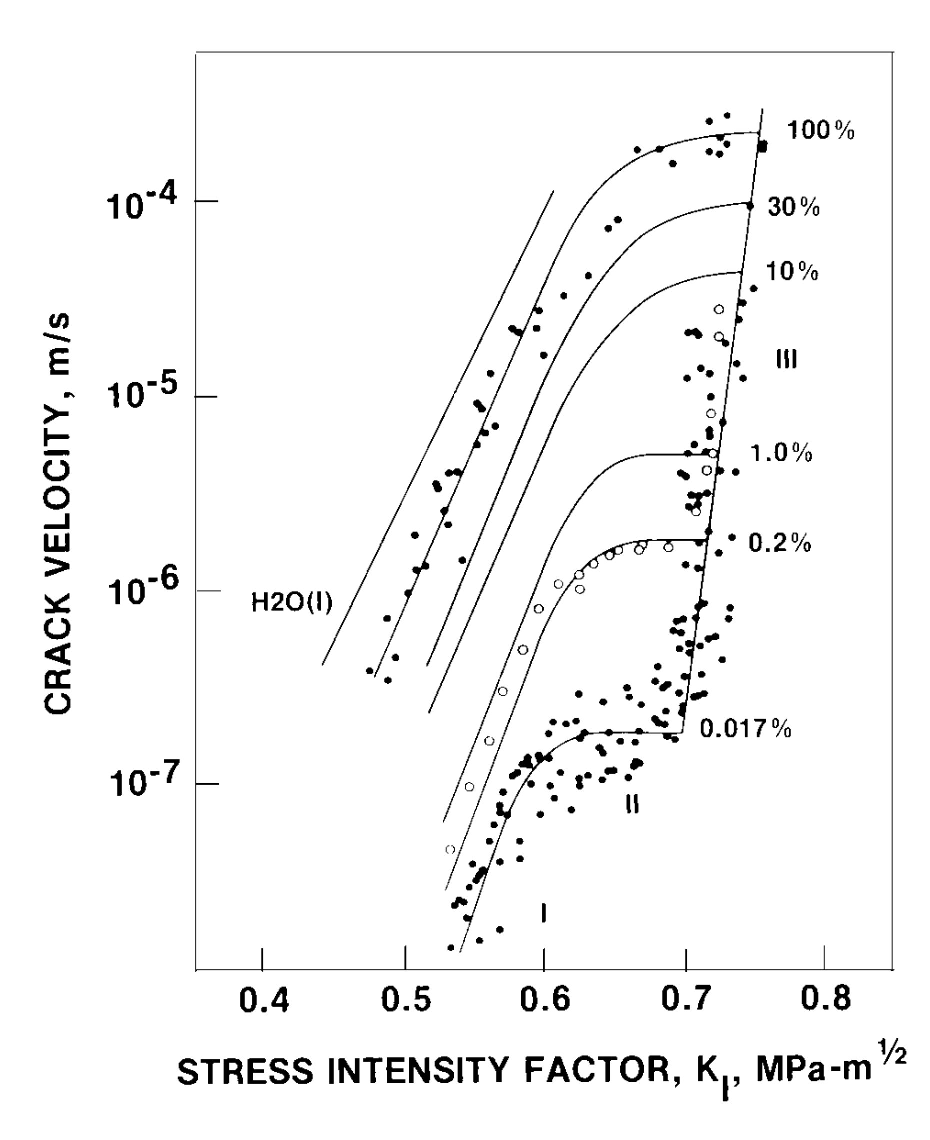
Water and stress enhance crack growth in glass

Regions I, II and III
"identify" behavior

For convenience, v-K_I relationship takes power law form :

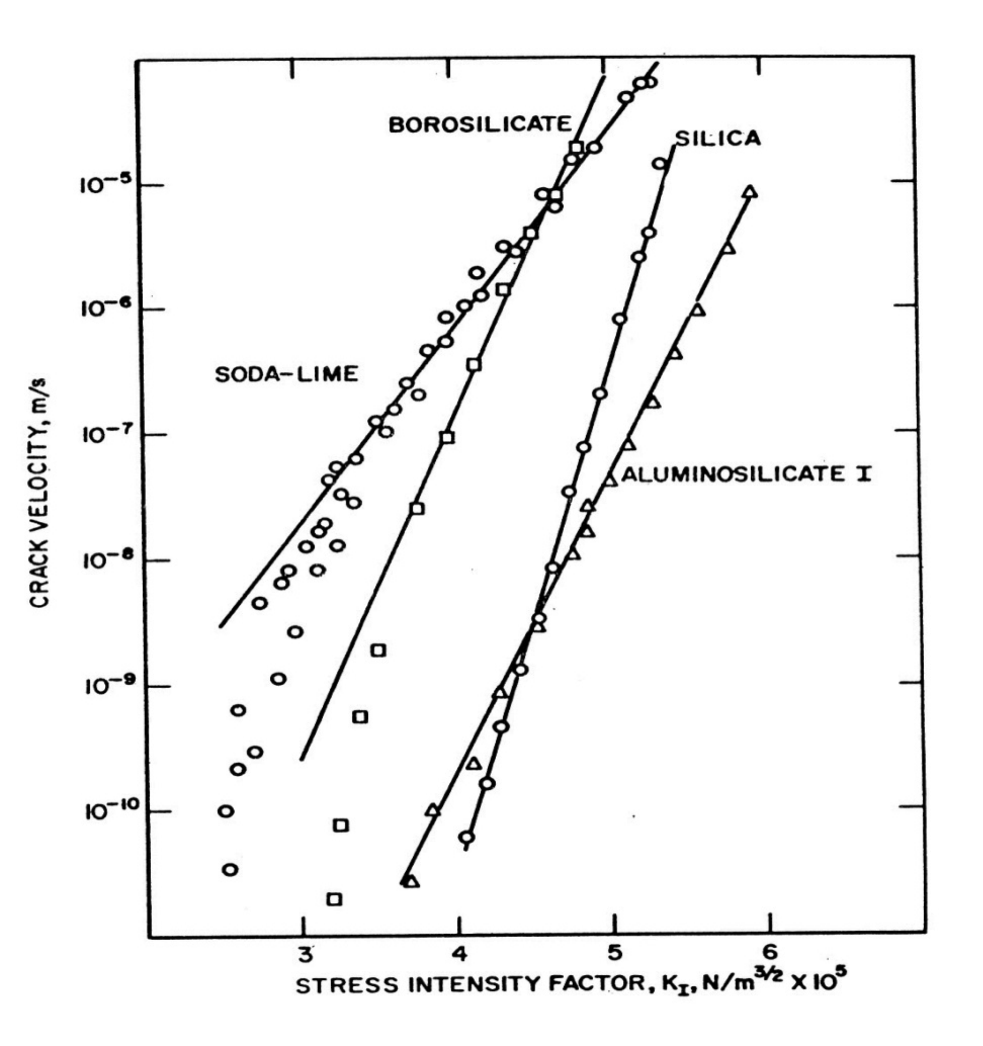
$$v = A K_I^n$$

n is called the "stress corrosion susceptibility parameter"



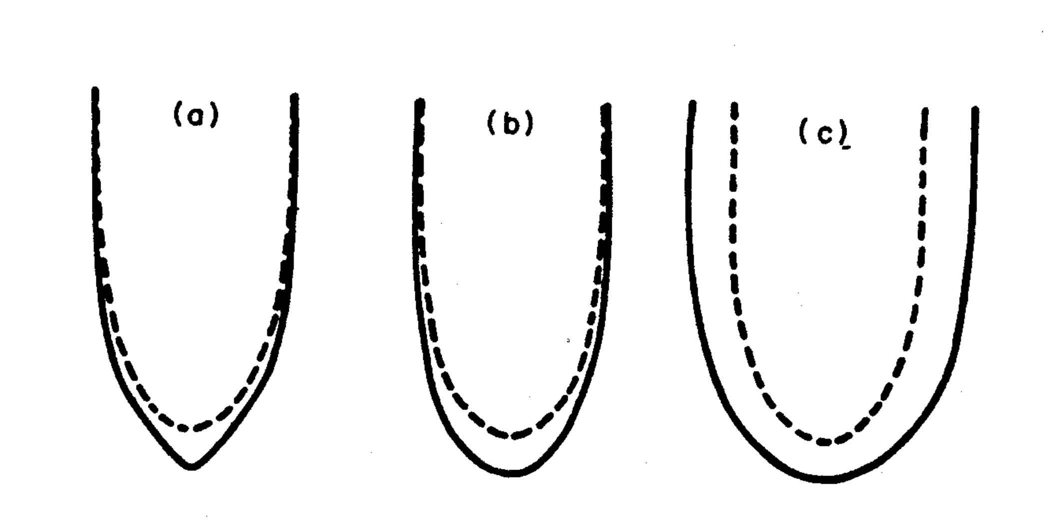
S.M. Wiederhorn, "Influence of Water Vapor on Crack Propagation in Soda-Lime Glass," *J. Am. Ceram. Soc.* **50** [8] 407-14 (1967).

Composition affects crack growth rate



Glasses with no alkali ions form straight lines on this kind of graph.

Crack sharpness is limited by the molecular structure of the glass



Similar to Figure 18-11

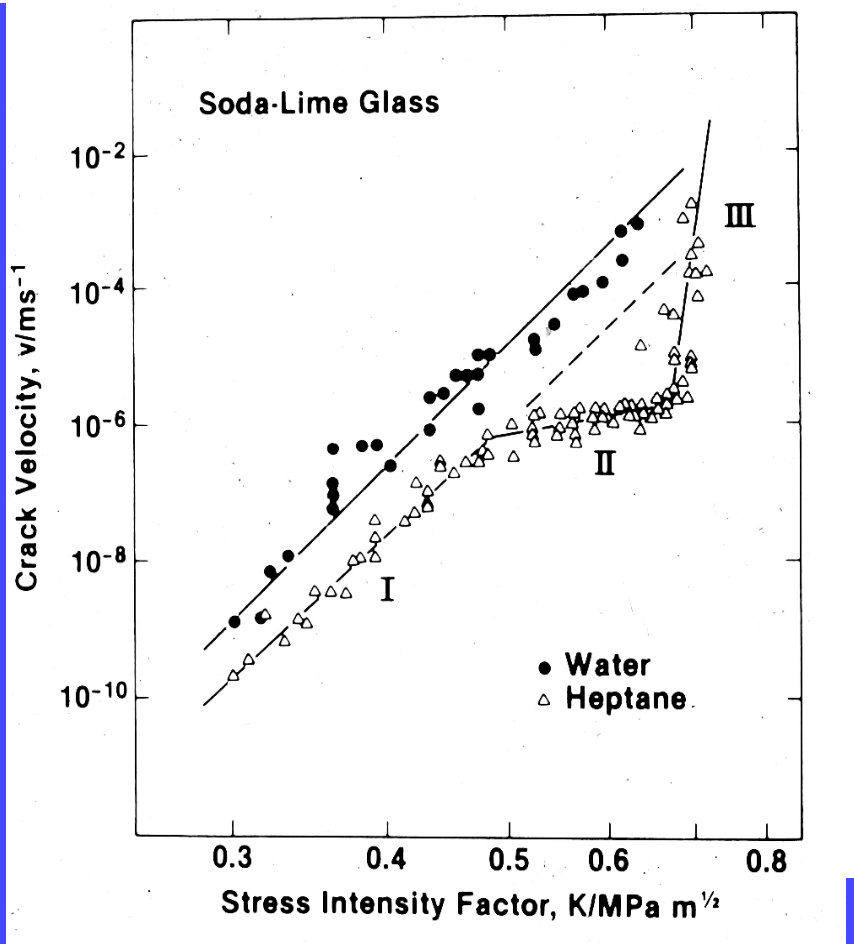
Change in crack tip geometry due to corrosion: (a) Flaw sharpening for stresses greater than the fatigue limit; (b) Constant flaw sharpness for stresses equal to the fatigue stress; (c) Flaw blunting for stresses below the fatigue limit.

.

T.-J. Chuang and E.R. Fuller, Jr. "Extended Charles-Hillig Theory for Stress Corrosion Cracking of Glass," *J. Am. Ceram. Soc.* 75[3] 540-45 (1992).

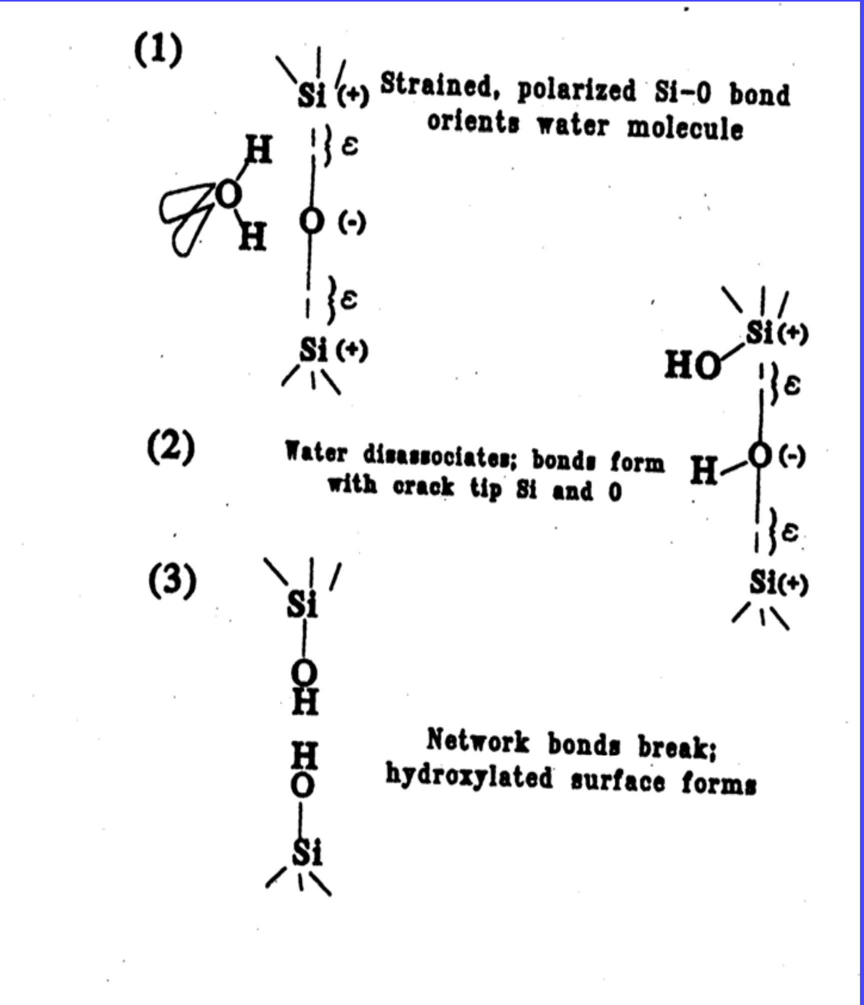
W.B. Hillig, "Model of effect of environmental attack on flaw growth kinetics of glass," *Int. J. Fract.* **143** 219-230 (2007)

Many alcohols do not affect crack growth – it is the water content in the alcohol!



The data for heptane was taken for a relative humidity of 50 %. The position of the curve for heptane is located at about 50% rh for air . Nothing in this figure about air.

There is a theory on how stress corrosion occurs in glass.



The three steps in the bond rupture process are:

1] A water molecule attaches to a bridging Si-O-Si bond at the crack tip. The water molecule is aligned by hydrogen bonding with the $O_{(bridging)}$ and interaction of the lone-pair orbitals from $O_{(water)}$ with Si.

2] A reaction occurs in which both proton transfer to the $O_{(br)}$ and electron transfer from the $O_{(w)}$ to the Si takes place simultaneously. During this step of the reaction the original bridging bond between $O_{(br)}$ and Si is destroyed.

3] Rupture of the hydrogen bond between $O_{(w)}$ and transferred hydrogen occurs to yield Si-O-H groups on each fracture surface.

T.A. Michalske and S.W. Freiman, "A Molecular Mechanism for Stress Corrosion in Vitreous Silica," *J. Am. Ceram. Soc.* **66**[4] 284-8 (1983).

There is a change in (fracture) surface energy with the presence of some environments

Table 18-1. Fracture Surface Energies of Some Glasses[a]

Glass	Environment	γ_f (J/m^2)
Pyrex	Air, 20 °C, 20% RH	4.7
	Air, 22 °C, 40% RH	4.0
	N_2 (gas), 27 °C, <0.1% RH	4.5-4.8
	N_2 (*l*), 77 K	4.7
	Water, 20 °C	2.5
Soda-lime (or float glass)	Air, 20 °C, 20% RH	3.9
	N_2 (*l*), 77 K	4.1
	N_2 (*l*), 77 K	4.5-4.6
	N_2 (gas), 27 °C, <1% RH	3.8-3.9
	Air, 22 °C, 40% RH	3.5
	Vacuum, 10^{-1} torr	5.0
Fused silica	N_2 (gas), 27 °C, <1% RH	4.3-4.4
	N_2 (*l*), 77 K	4.6
	Air, 22 °C, 40% RH	3.7
Aluminosilicate	N_2 (gas), 27 °C, <1% RH	4.6-4.7
	N_2 (*l*) 77 K	5.2
	Air, 22 °C, 40% RH	3.7

[a] After Ref. 26, Table IV. Reproduced with the permission of the Academic Press

Stress Corrosion Susceptibility depends on composition and structure

Material	n	K_{IC} ($MPam^{1/2}$)
Fused Silica	30	0.75
SLS glass	5-15	0.7
Pyrex (B_2O_3)	10	0.77
aluminosilicate	10-15	0.85
Lead silicate	5-10	0.63
Chalcogenides	5 -15 ?	0.2 – 0.3

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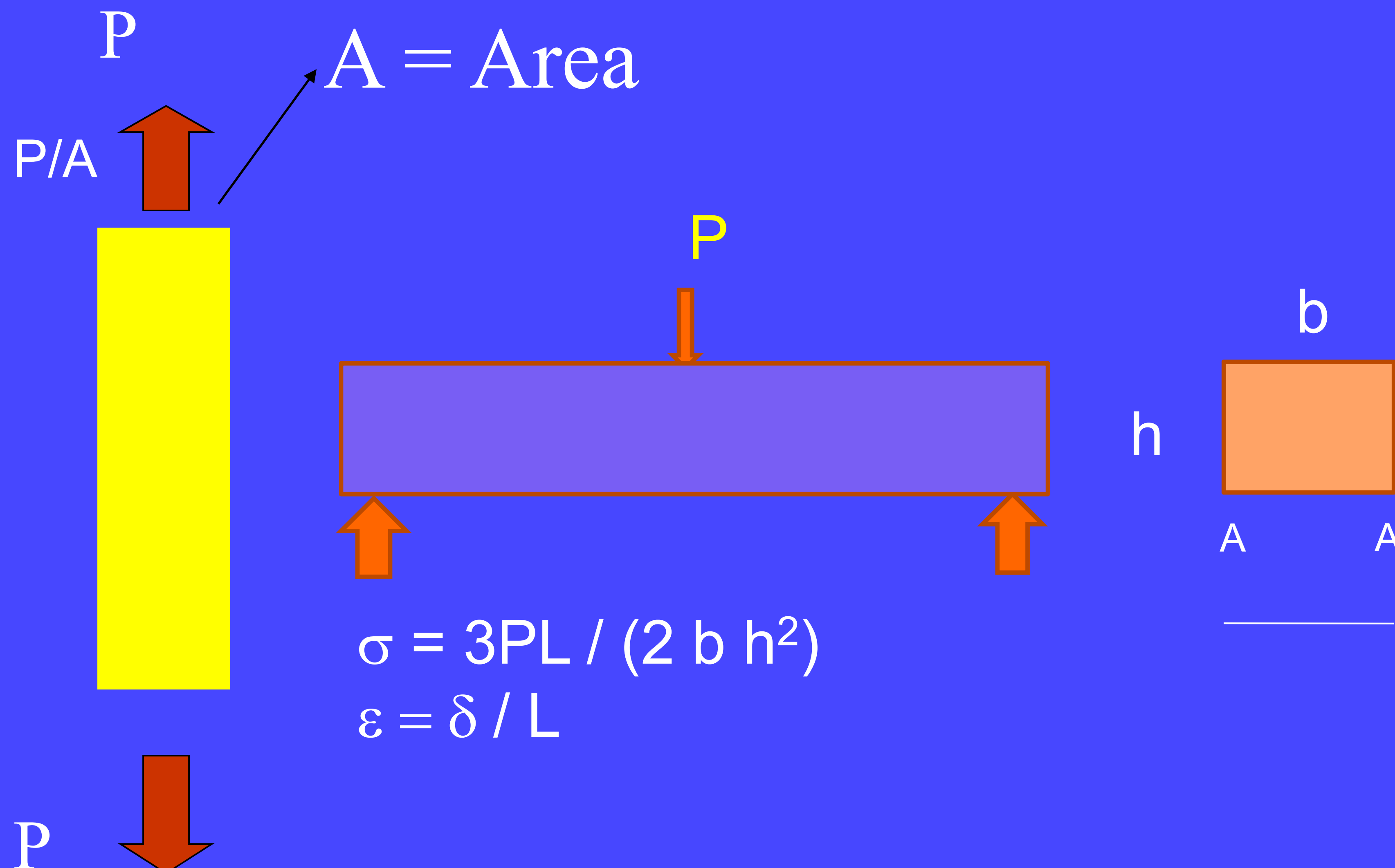
  - Stress Corrosion
  - Fracture Statistics

*A. Varshneya, "Fundamentals of Inorganic Glasses", Society of Glass Technology (2006)

For same size bar, which would have the greater strength? Why?



$\sigma = P/A$

$\sigma = 3PL / (2\ b\ h^2)$

$\varepsilon = \delta / L$

Weibull statistics is a "weakest link" theory

$P = 1 - \exp[-R]$

$R = \int_v [(\sigma - \sigma_u) / s_0]^m \, dv$ for $\sigma > \sigma_u$

$P = 0$ for $\sigma < \sigma_u$

In many cases, we assume $\sigma_u = 0$ and

$R = Y_v \, v \, [\sigma/\sigma_0]^m$

m is Weibull modulus

$Y_v \, v$ is effective volume

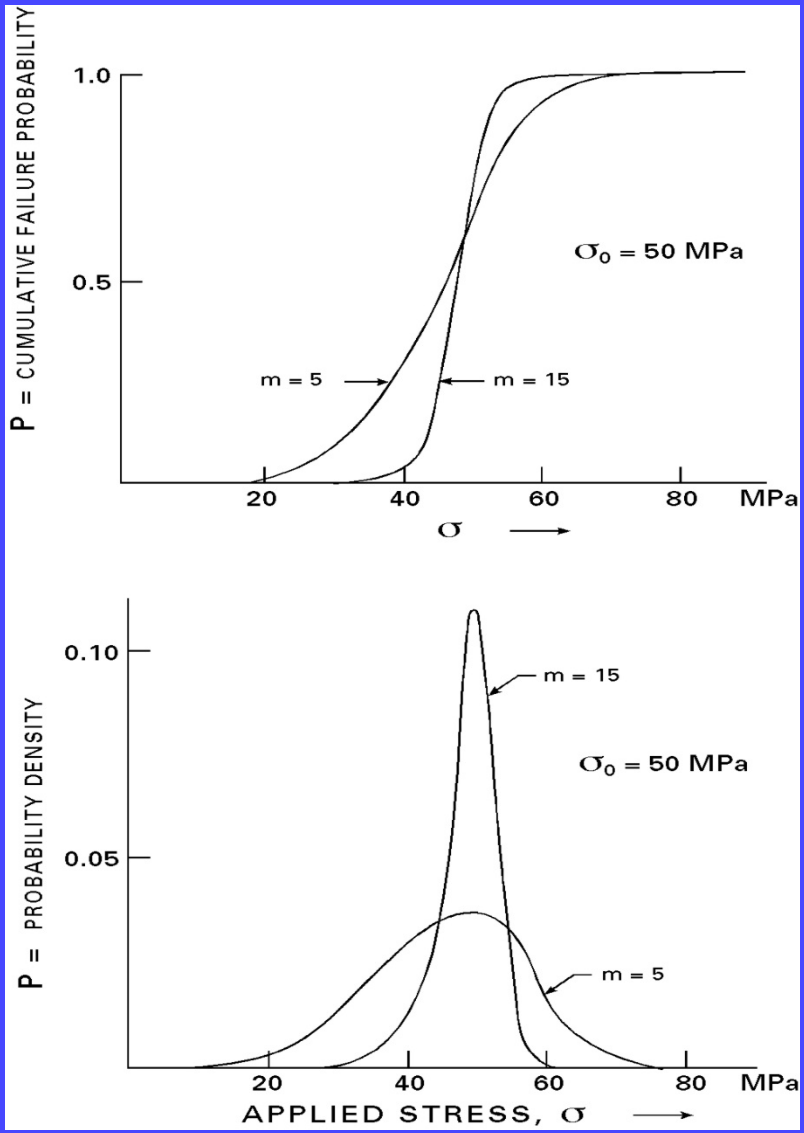


Fig. 18-18

The effective volume varies with loading

For tension,

$Y\nu = 1$

For pure bending,

$Y\nu = 1/ [2(m + 1)]$

For 3-point flexure,

$Y\nu = 1/ [2(m + 1)^2]$

We also can use an effective surface area if only surface flaws are considered.

and for 4-point loading

$$Y_v = [m(L_i/L_o)+1]/[2(m+1)^2]$$

where L_i is the inner span and L_o is the outer span.

If the flaws are distributed on the surface only, then eq.(18.27) m modified to

$$R = \int_A [\sigma/\sigma_o]^m \; dA \qquad ($$

$$= Y_s A[\sigma/\sigma_0]^m$$

where $Y_s A$ may be called **effective surface area**. Again, by simple int tion, it may be shown that

$Y_s = 1$ for uniform tension,

$= [(w/h)+\{1/(m+1)\}]/2[1+(w/h)]$ for pure bending,

$= [(w/h)+\{1/(m+1)\}]/2[\{1+(w/h)\}\{m+1\}]$ for 3-point loading,

$= [m(L_i/L_o)+1][(w/h)+\{1/(m+1)\}]/2[\{1+(w/h)\}\{m+1\}]$ for 4-point loac

It may be recognized that Y and v are material and loading const which can be lumped back into a redefined σ_0 (with units of stress) that a simplified form of the Weibull distribution is

$$P = 1 - \exp\{-[\sigma/\sigma_0]^m\} \qquad (18$$

The derivative of the Weibull distribution $dP/d\sigma$ gives the probab density function p which is the probability that the specimen will fail wi σ and $\sigma + d\sigma$, i.e. the strength distribution

$$p = dP/d\sigma = [m/\sigma_0][\sigma/\sigma_0]^{m-1}\exp[-(\sigma/\sigma_0)^m] \qquad (18$$

Plots of equations (18.29) and (18.30) are shown in Fig. 18-18. Note skewness of the strength distribution.

Taking logarithms of the survival probability $(1-P)$ twice, we get fr eq. (18.29):

$$\ln\ln[1/(1-P)] = m\ln\sigma + \ln[1/\sigma_0{}^m] \qquad (18$$

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One could rewrite the Weibull distribution to include the effects of time under stress (which represents static fatigue) and the specimen size (which reflects volume or surface effects as above) as:

$$P = 1 - \exp\{-[\sigma/\sigma_0]^m [t/t_0]^r [L/L_o]^q\} \quad (18.32)$$

Linearized version is easiest to understand:

$$\ln\ln[1/(1-P)] = m\ln(\sigma) - m\ln(\sigma_0)$$

The slope , m, is a measure of the scatter of the data

A small value, e.g., 2-10, is an indication of great scatter. A large value, such as 30-99 shows little scatter.

σ_0 is the Weibull effective stress at ~ 63% failure probability.

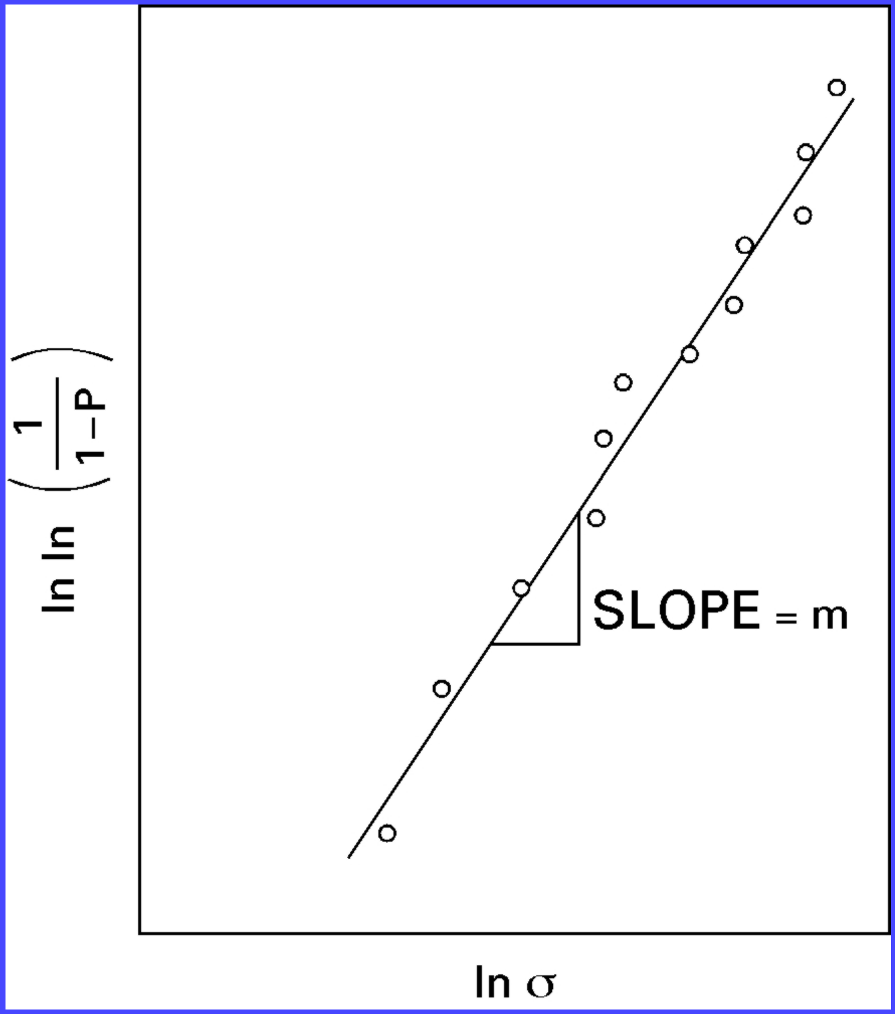

Fig. 18-19

Lifetime predictions combine fracture mechanics, stress corrosion and probability.

$t_{min}=2[\sigma_p/\sigma_a]^{n-2} / [K_{IC}^{n-2}\ \sigma_a^2\ A\ Y^2(n-2)]$

σ_p is the proof stress, i.e., a pre-applied stress greater than expected in service.

log t_{min} vs. log σ_a results in a proof test diagram as a function of proof stress ratio.

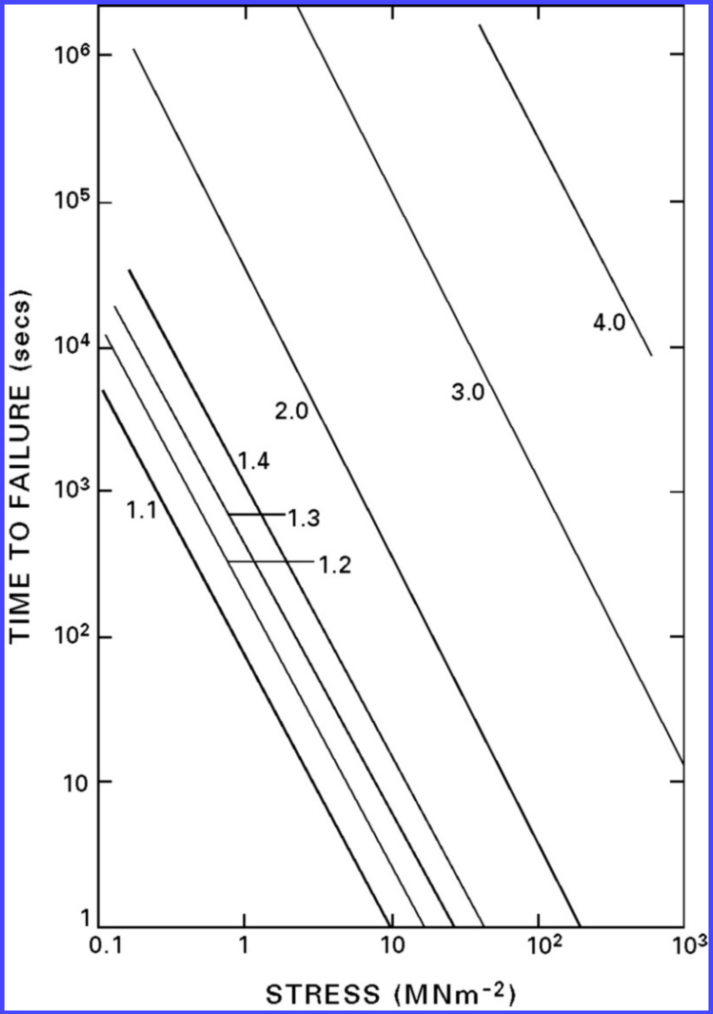

Fig. 18-20

Summary

Strength is the applied stress at failure

Glass fails in tension, i.e., when the maximum principal stress is perpendicular to the plane of the crack. However, it may be loaded in many directions (Mode I, II, III)

Fracture mechanics governs the failure of glass. $K_{IC} = Y \sigma_f (c)^{0.5}$.
[Recall the work of Inglis, Griffith and Irwin]

Fracture occurs due to the application of the greatest stress in the region with the largest crack. Therefore there is a statistical nature to the strength of materials, but the fracture toughness of a glass will be constant.

The environment can decrease the strength of glass due to a stress enhanced, chemical reaction at the tip of the crack. Thus, there can be a time dependence to failure. In some glasses, a **stress corrosion fatigue limit** can exist. That is, below a certain tensile stress value, slow crack growth will not occur.

Summary (cont'd)

The occurrence of cracks in glass is a probabilistic event; therefore, strength is probabilistic. A reasonable theory that can be used to model the statistical nature of strength is called the Weibull distribution and is based on a weakest link argument.

Fractography, i.e., the examination of fracture surfaces, shows characteristic features known as mirror, mist and hackle. These regions can be used to identify the origin of fracture, the stress at fracture and the nature of the failure.